Machine Drive System

Components of variable-speed system :-

① Power converters (Drive circuits)

- Controlled Rectifiers (AC-DC)

- Choppers (DC-DC)
  "Buck converter - Step down DC-DC converter"

- Inverters (DC-AC)
  - 6 step inverter
  - Voltage source inverter (VSI) or 2-level inverter

- AC Controllers (AC-AC)

② Electrical Machines

- DC Machine (series, shunt, compounded, separately excited DC motors)
- AC Machine

(Induction motors + synchronous motors)

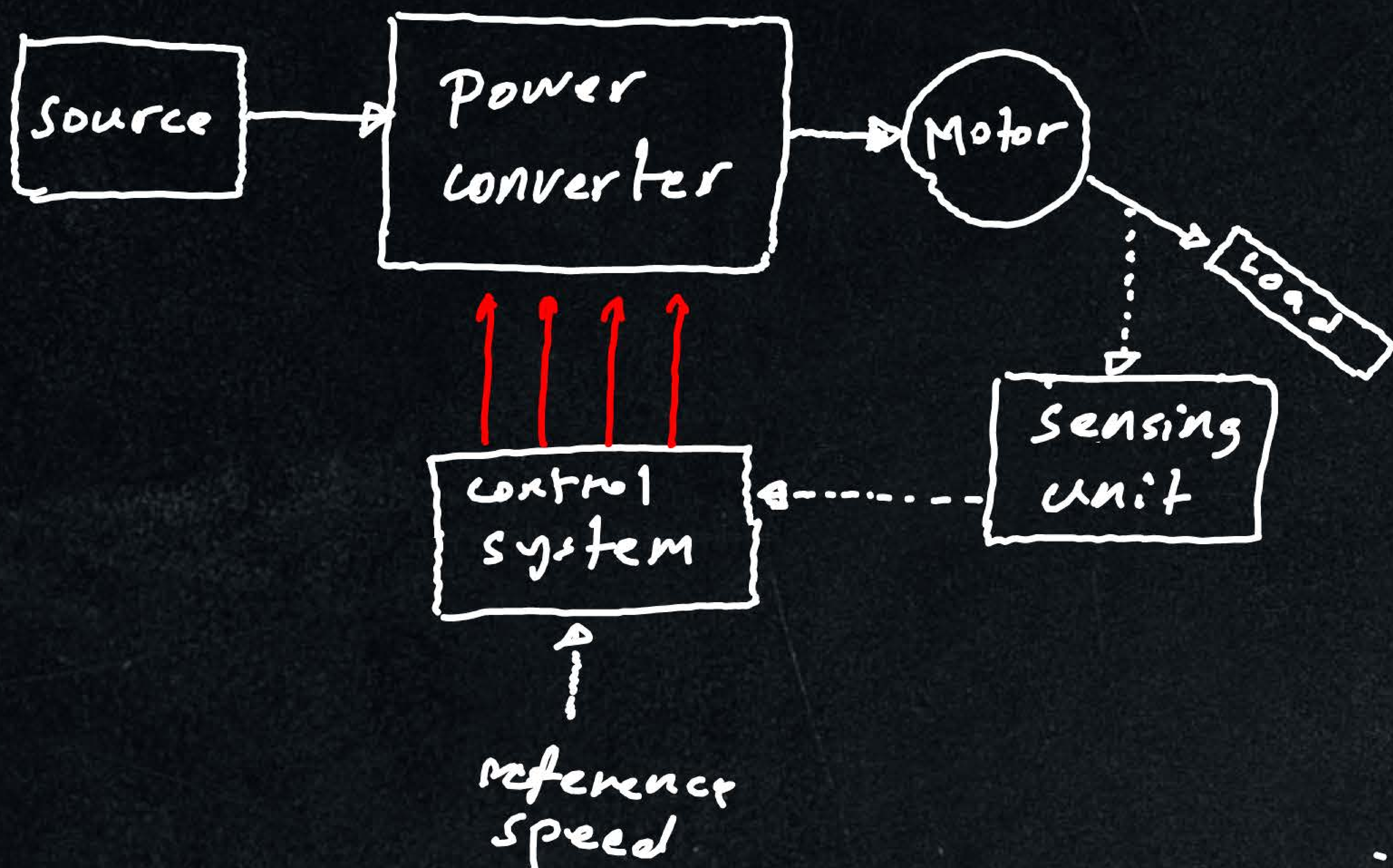
③ Control system

It matches the motor and power converter to meet the load requirement.

④ Load

.

Block diagram of machine drive system



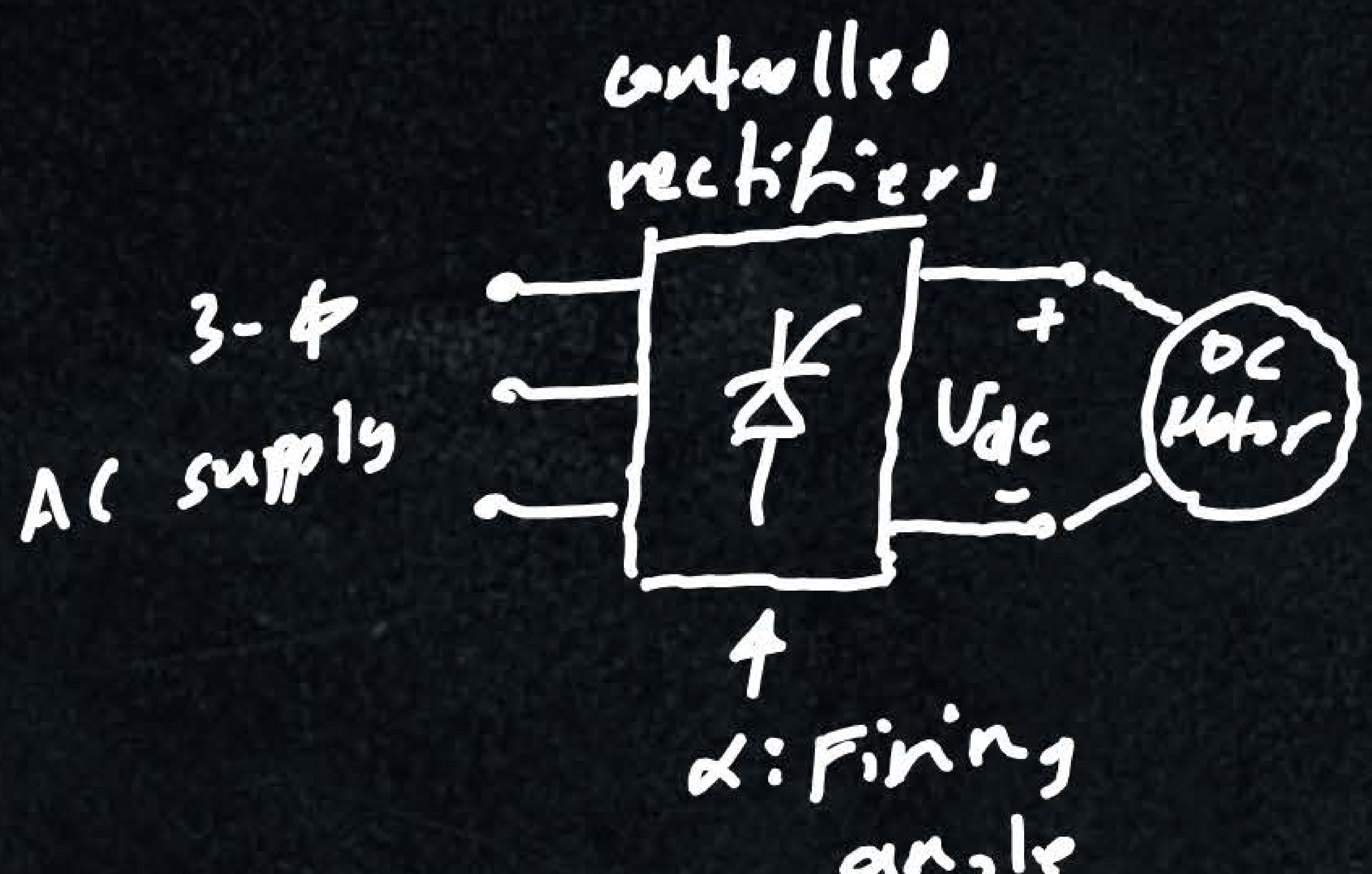
① power converters

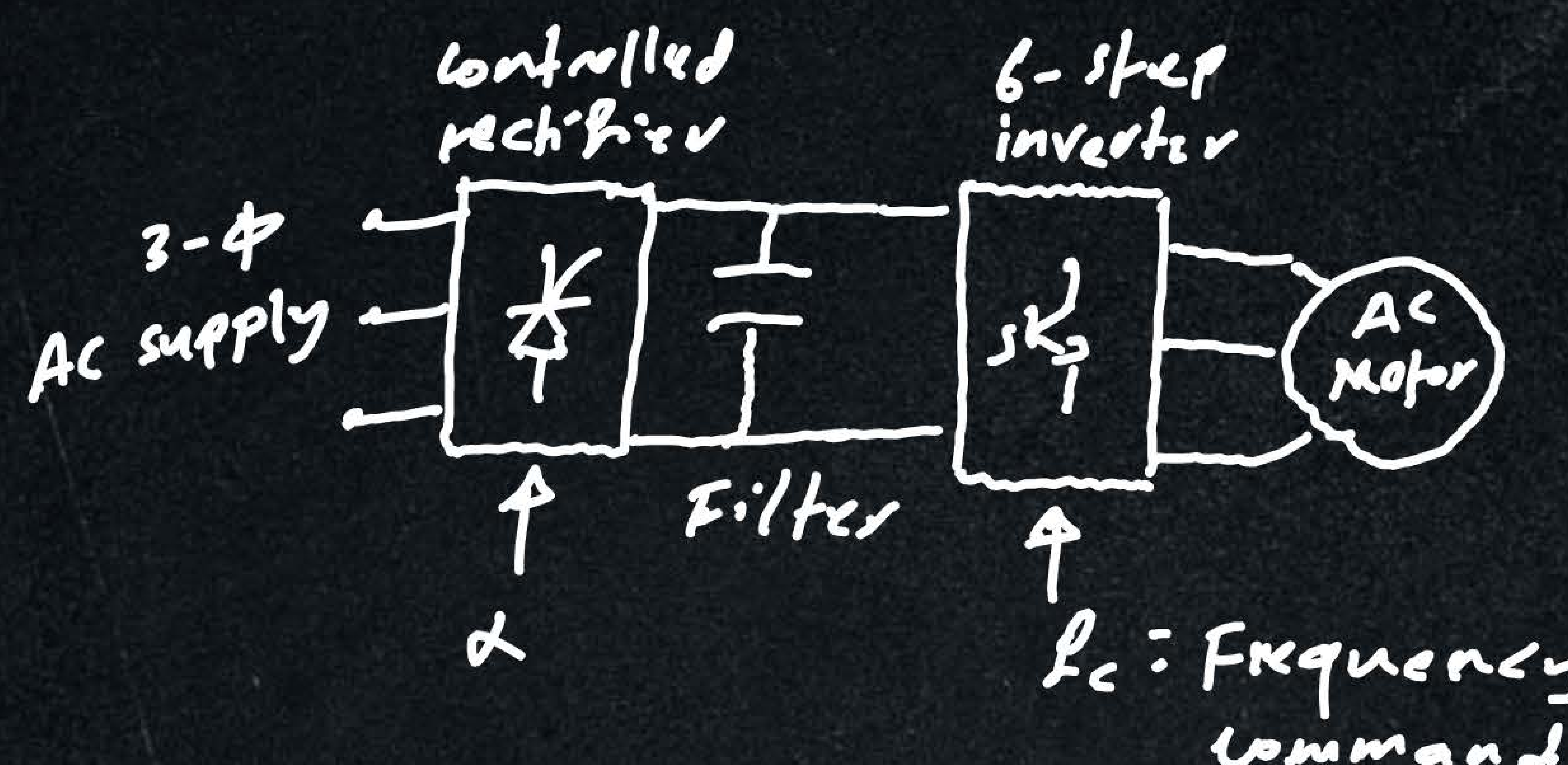
1.1) Controlled Rectifiers

input : 1-ϕ or 3-ϕ AC main supply

output : Variable DC voltage for the control of DC machine

or sometimes: input DC supply to 6-step inverter in the case of AC Machine

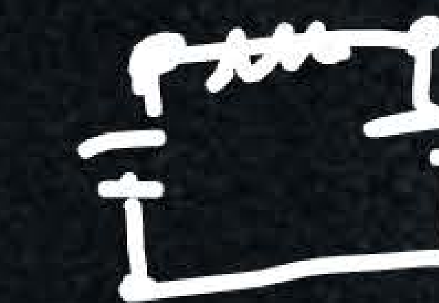


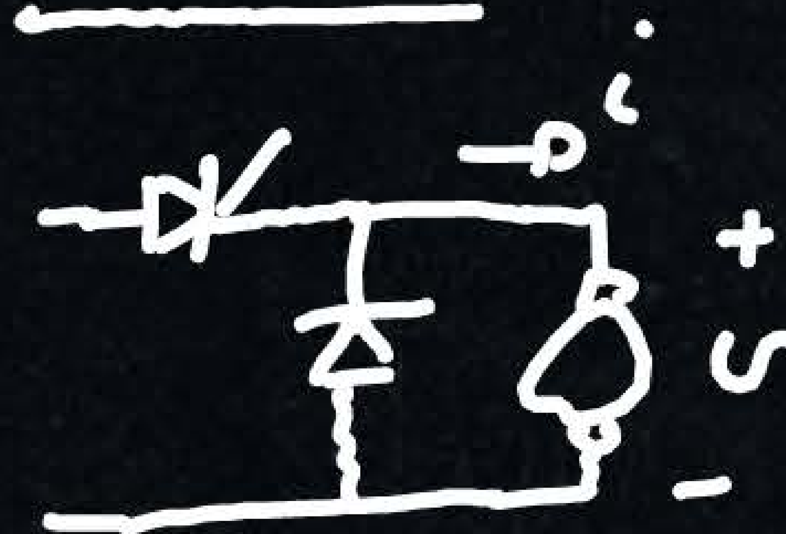
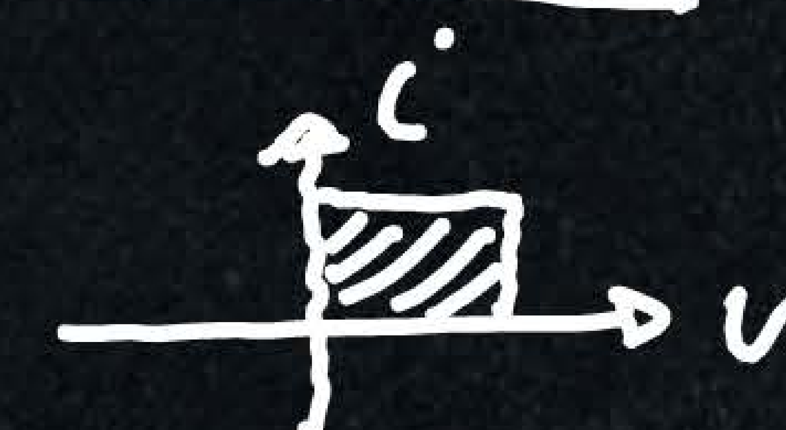
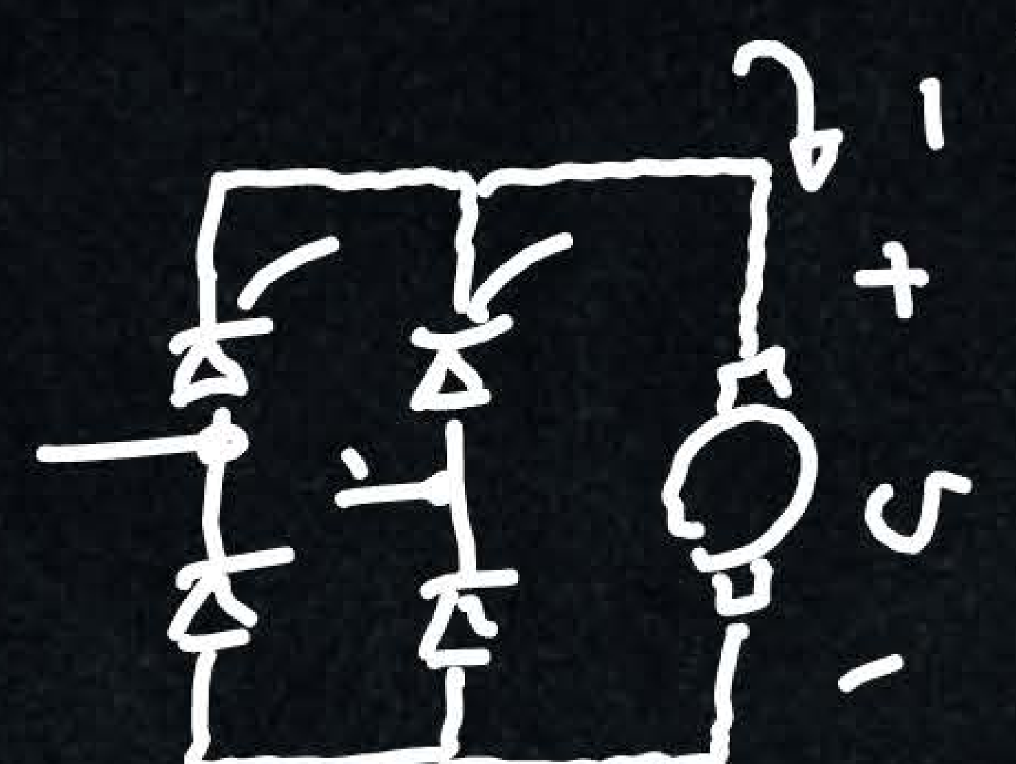
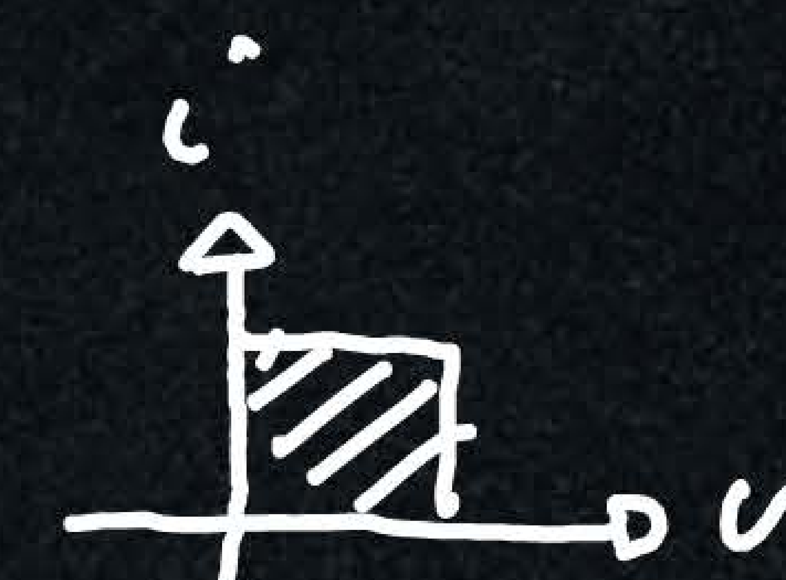
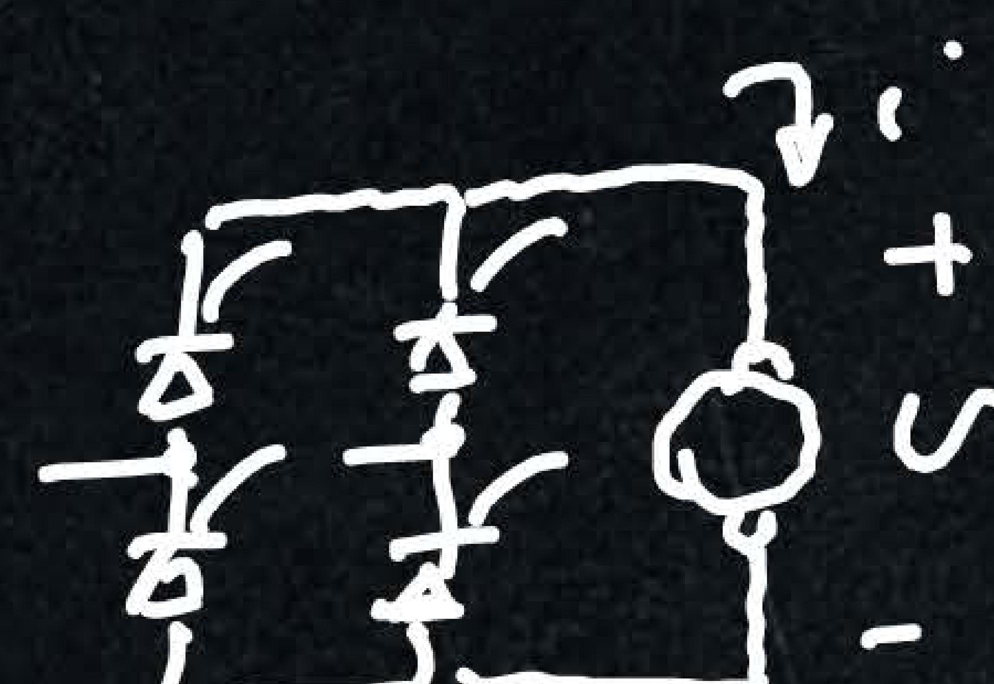
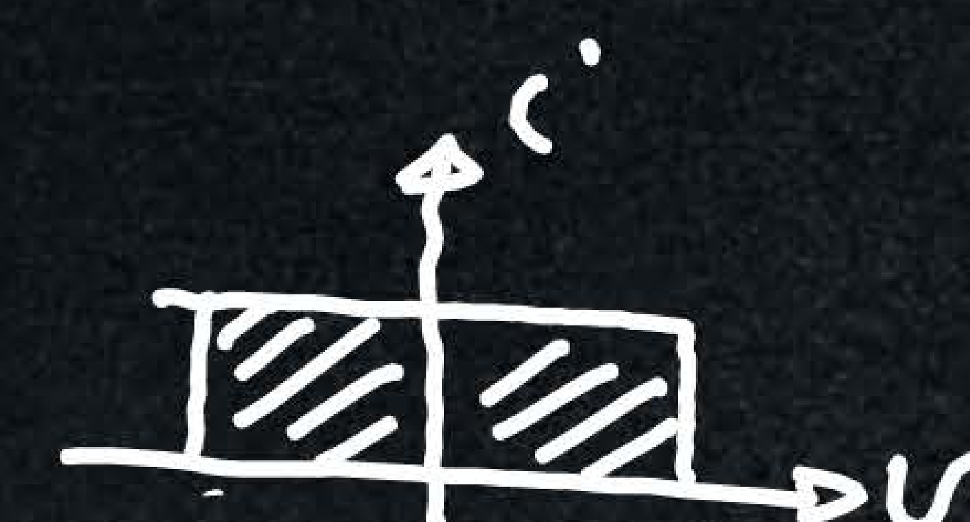
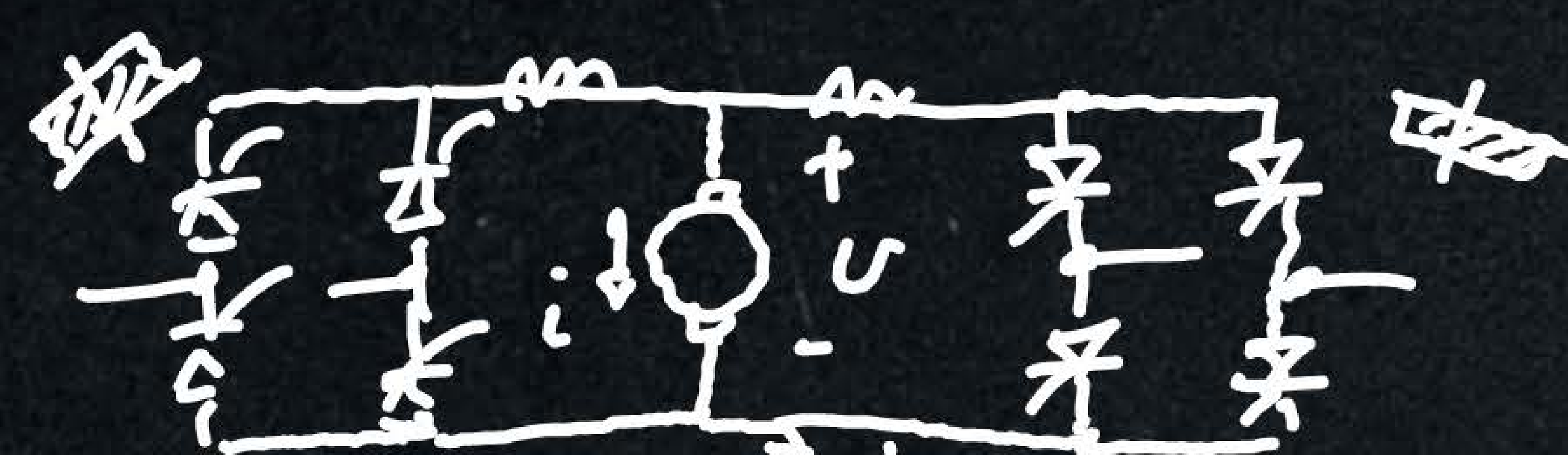
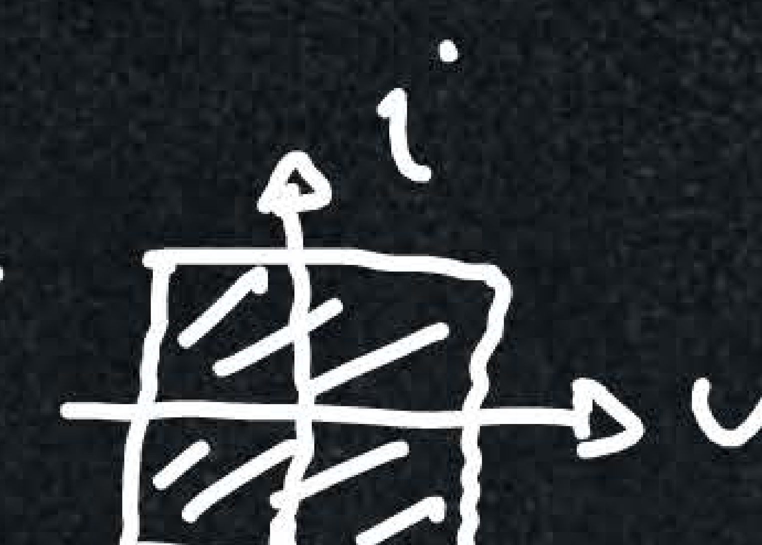


single-phase

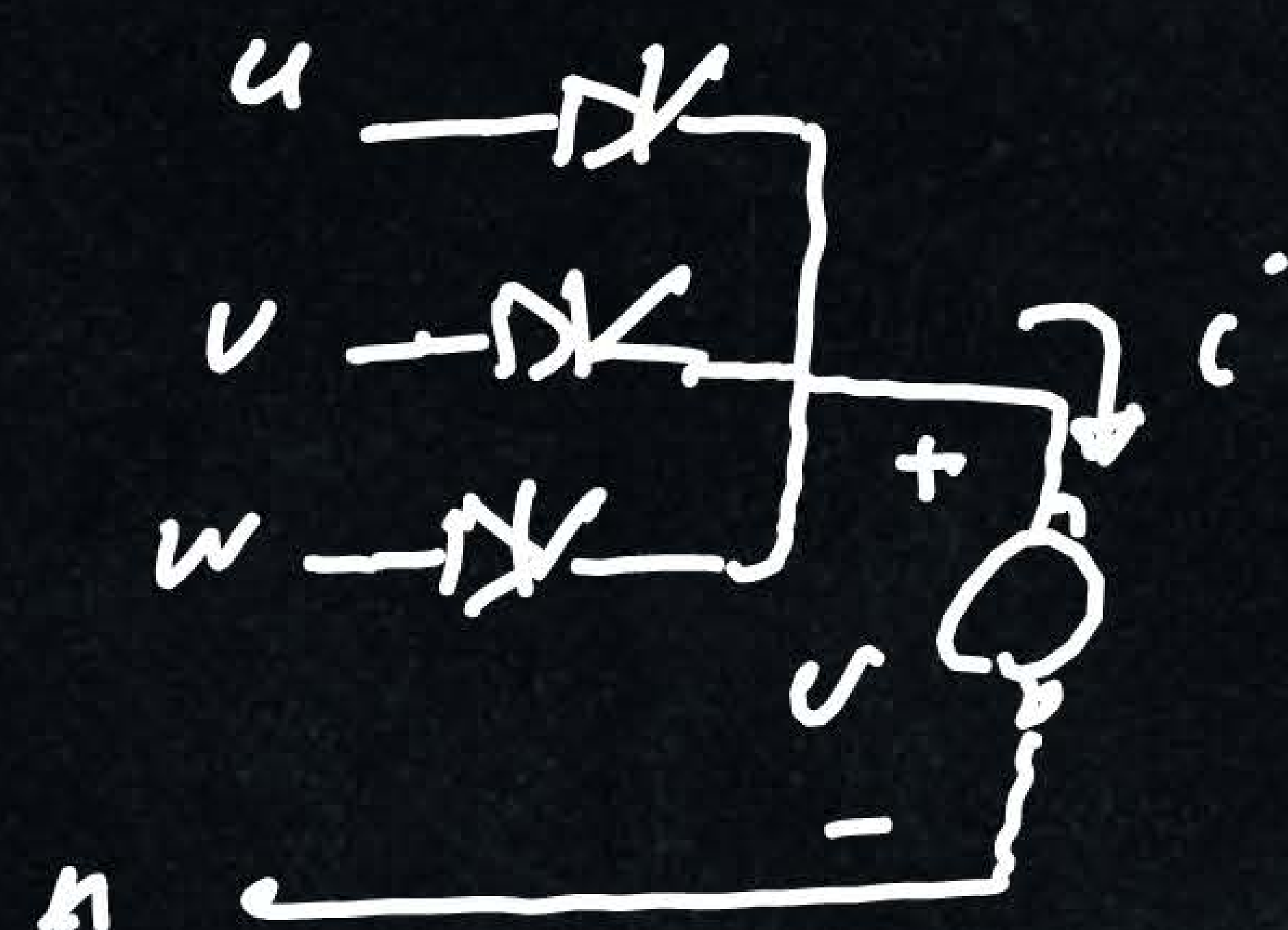
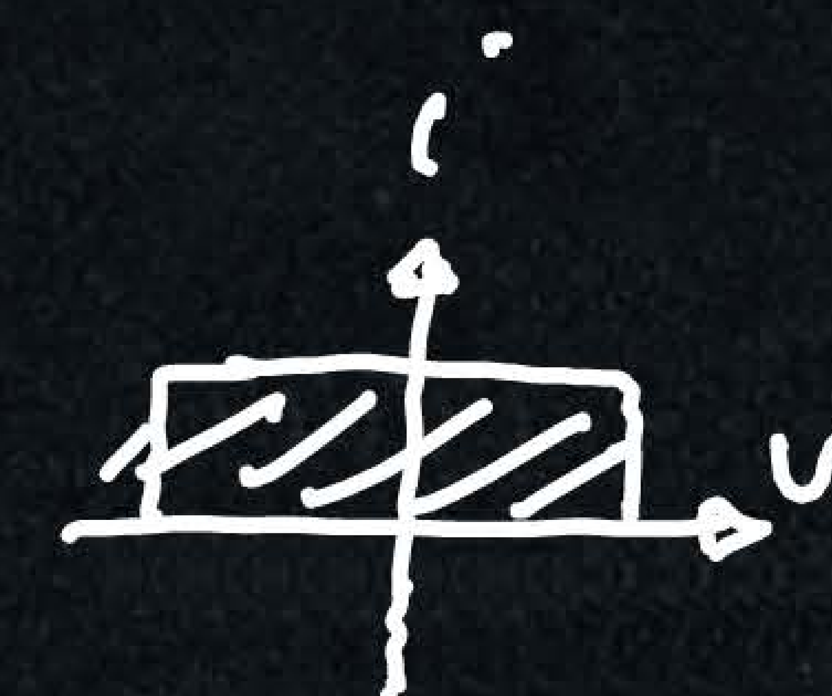
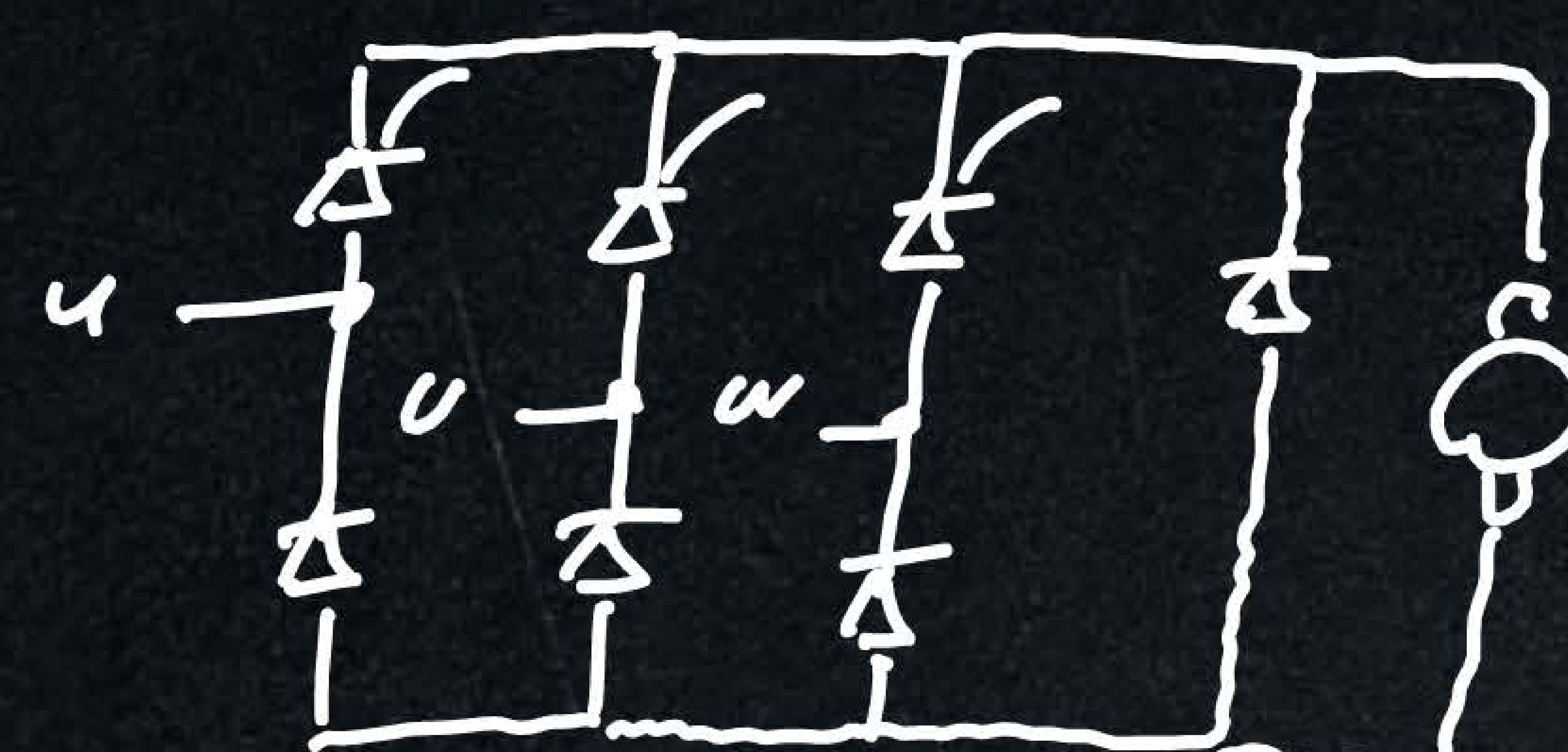
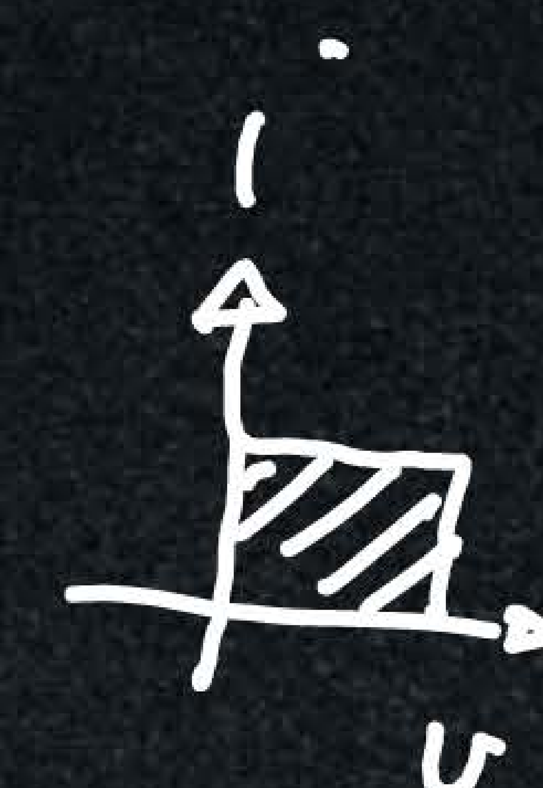
$K\phi i$

$$T_{el} - T_2 = J\frac{d\omega_m}{dt}$$



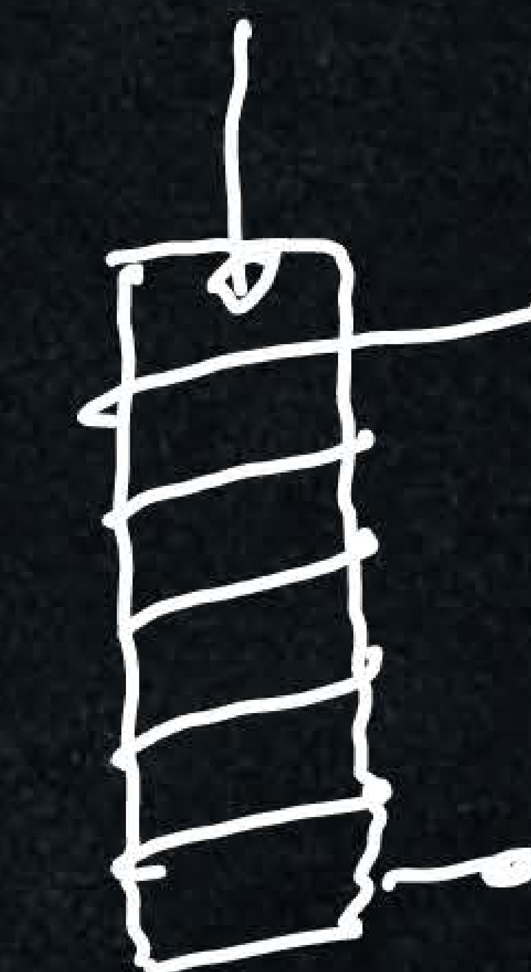
Type	circuit	operation	Average voltage
Half-wave			$\frac{V_m}{2\pi}(1+\cos\alpha)$
semi-converter			$\frac{V_m}{\pi}(1+\cos\alpha)$
Full-wave			$\frac{2V_m}{\pi}\cos\alpha$
Dual converter			$\frac{2V_m}{\pi}\cos\alpha$

Three-phase

Type	circuit	operation	Average voltage
Half-wave			$\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$ V_m: peak of L-N phase voltage
semi-converter			$\frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$

induction motor

$\Phi = \Phi_m \sin(\omega t + \theta)$



$+$

$e_{ind} \propto \frac{d\Phi}{dt}$

$-$

induced voltage

$e_{ind} = N \frac{d\Phi}{dt}$ "Faraday's law"

$\Phi = \Phi_m \sin(\omega t + \theta)$

↳ ω electric radian frequency

$e_{ind} = \underbrace{N \omega \Phi_m}_{\text{peak}} \cos(\omega t + \theta)$

$e_{ind,\,peak} = N \Phi_m \omega = V$

$\Phi_m = \frac{V}{N\omega}$

$$\boxed{\underset{\text{peak}}{\Phi_m} \propto \frac{V}{\omega}}$$

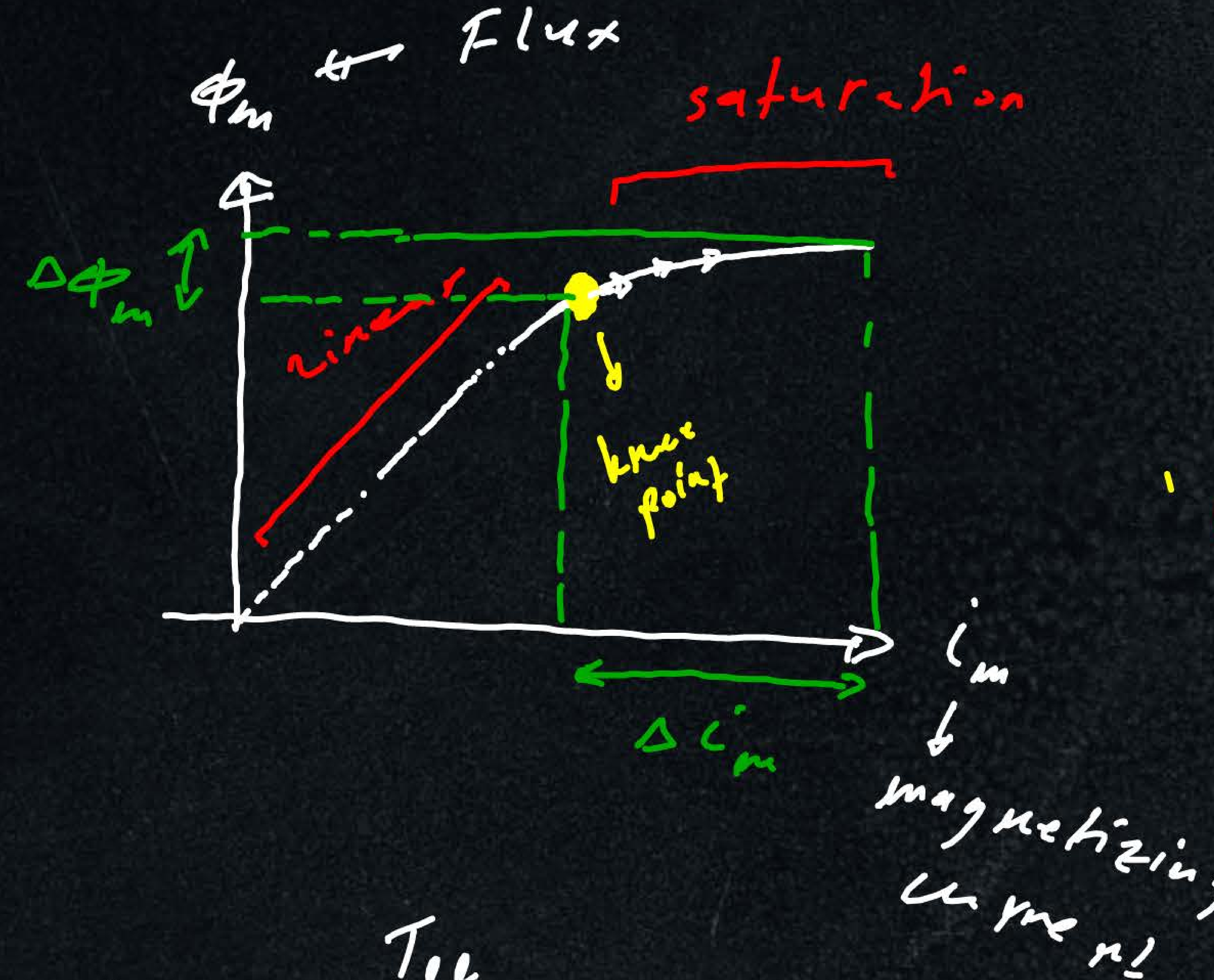
Magnetizing curve

$$\phi_m \propto \frac{V}{\omega}$$

$$n_s = \frac{120}{P} f_e$$

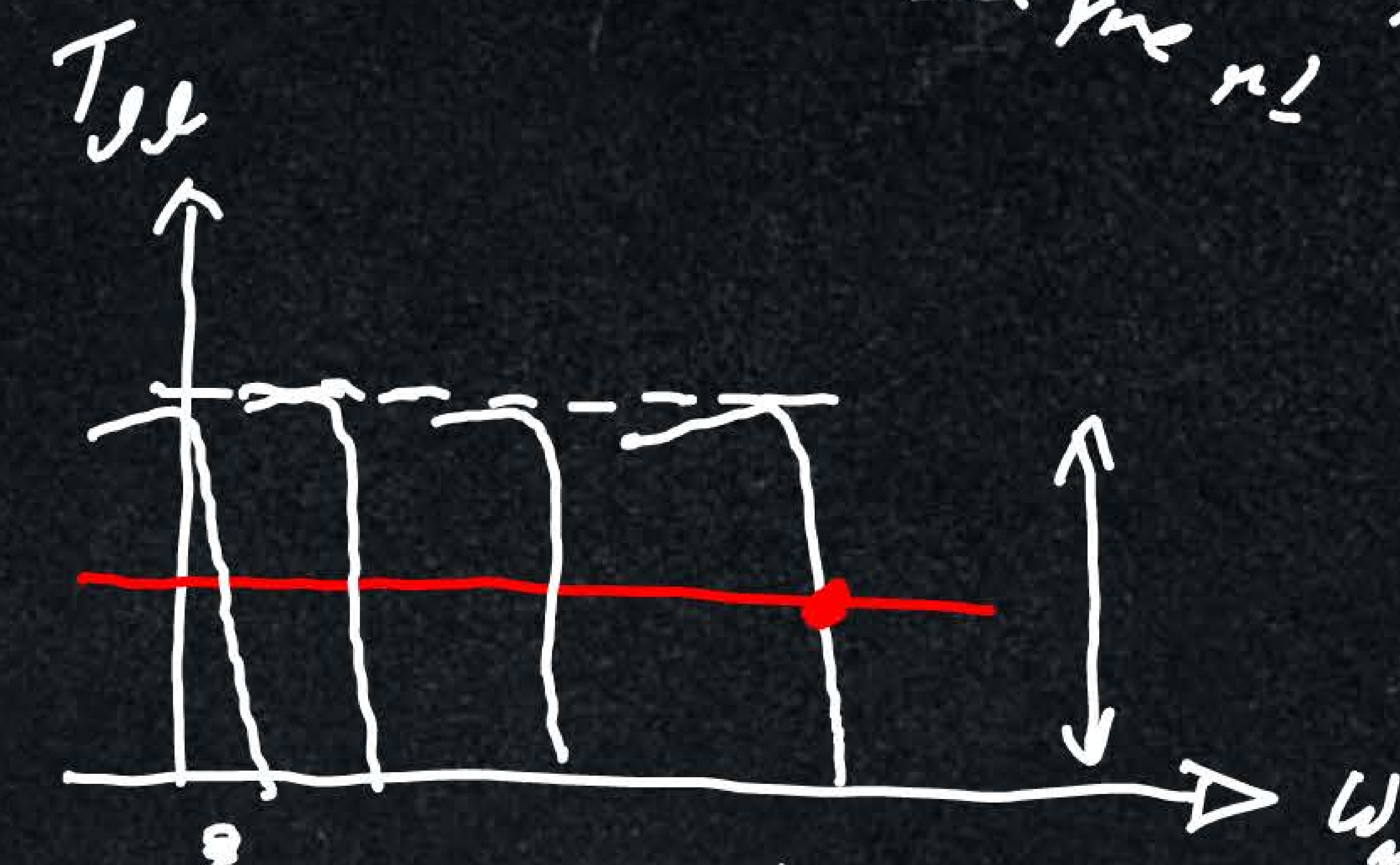
synchronous speed

$$f_e = \frac{\omega}{2\pi}$$



Flux current

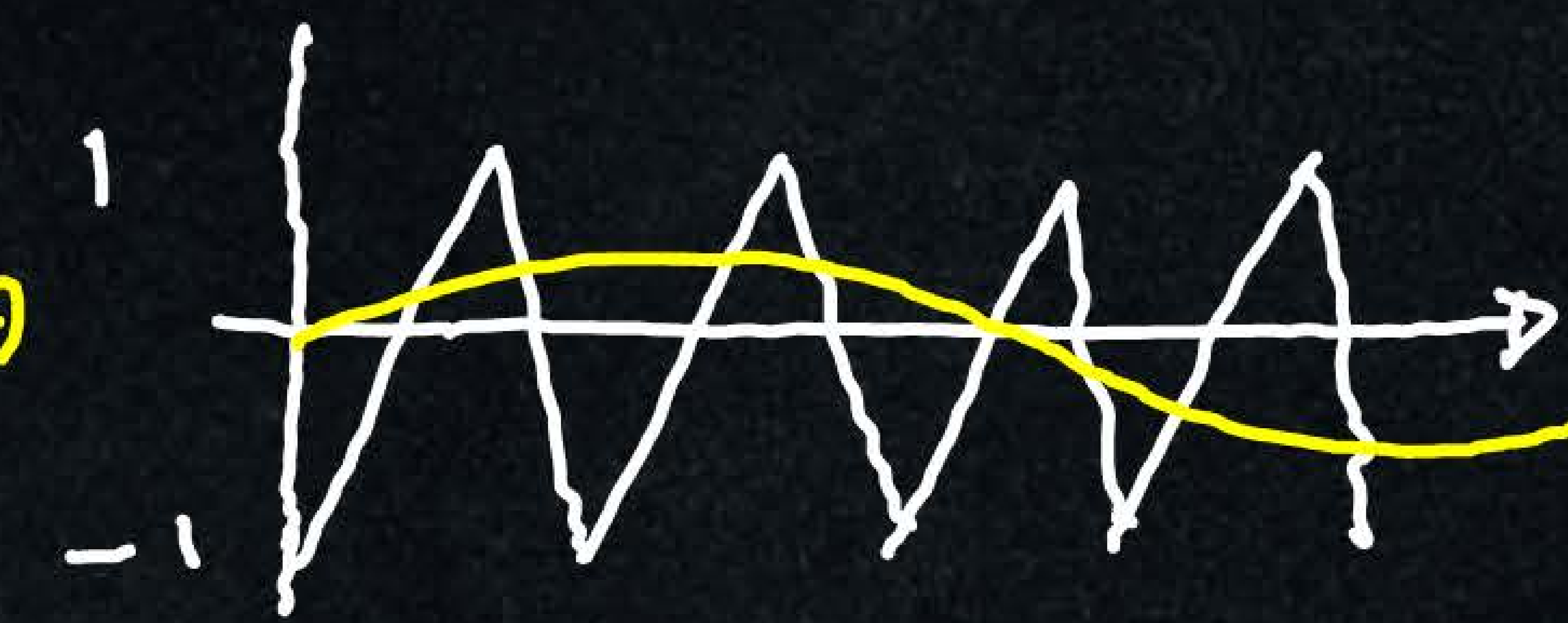
$$\underbrace{T_{el}}_{\text{rated torque}} = K \phi i$$



Control system for DC motor

modulating waves

$$m_a = M \sin(\omega t)$$



$$m_b = M \sin(\omega t \pm 120^\circ) \checkmark$$

$$m_c = M \sin(\omega t \mp 120^\circ) \checkmark$$

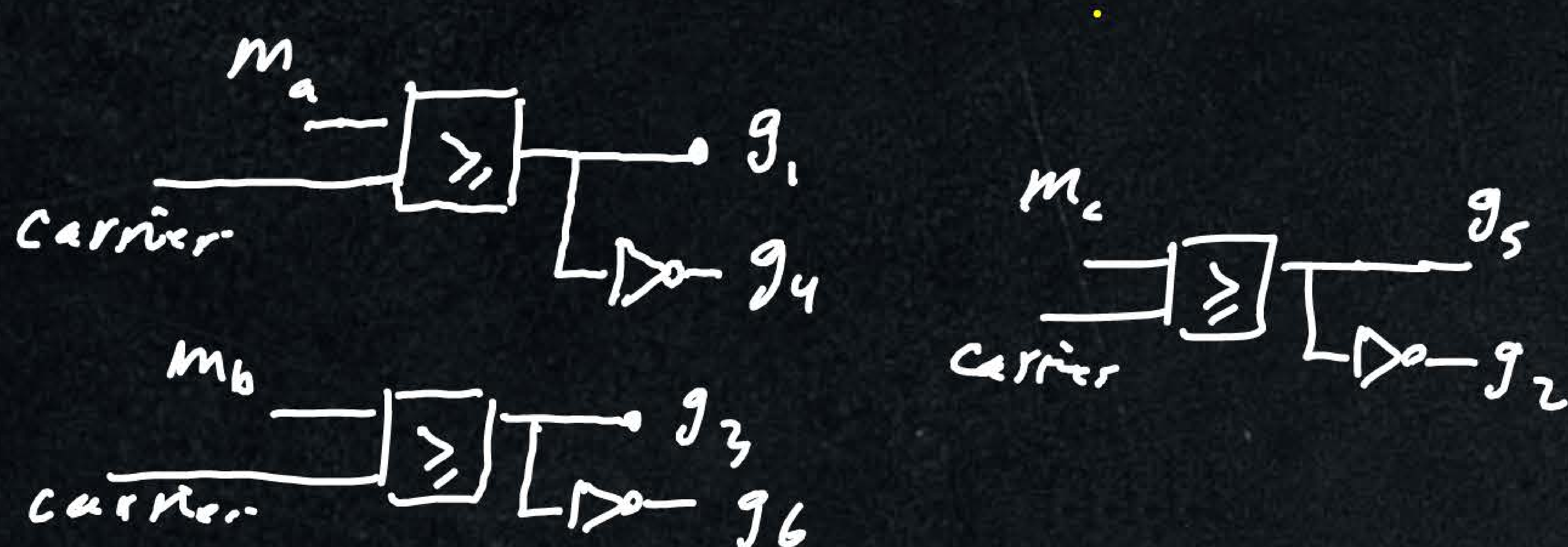
$$V_{L-N,\,peak} = \underset{\uparrow}{M} \frac{V_s}{2}$$

M: Modulation index

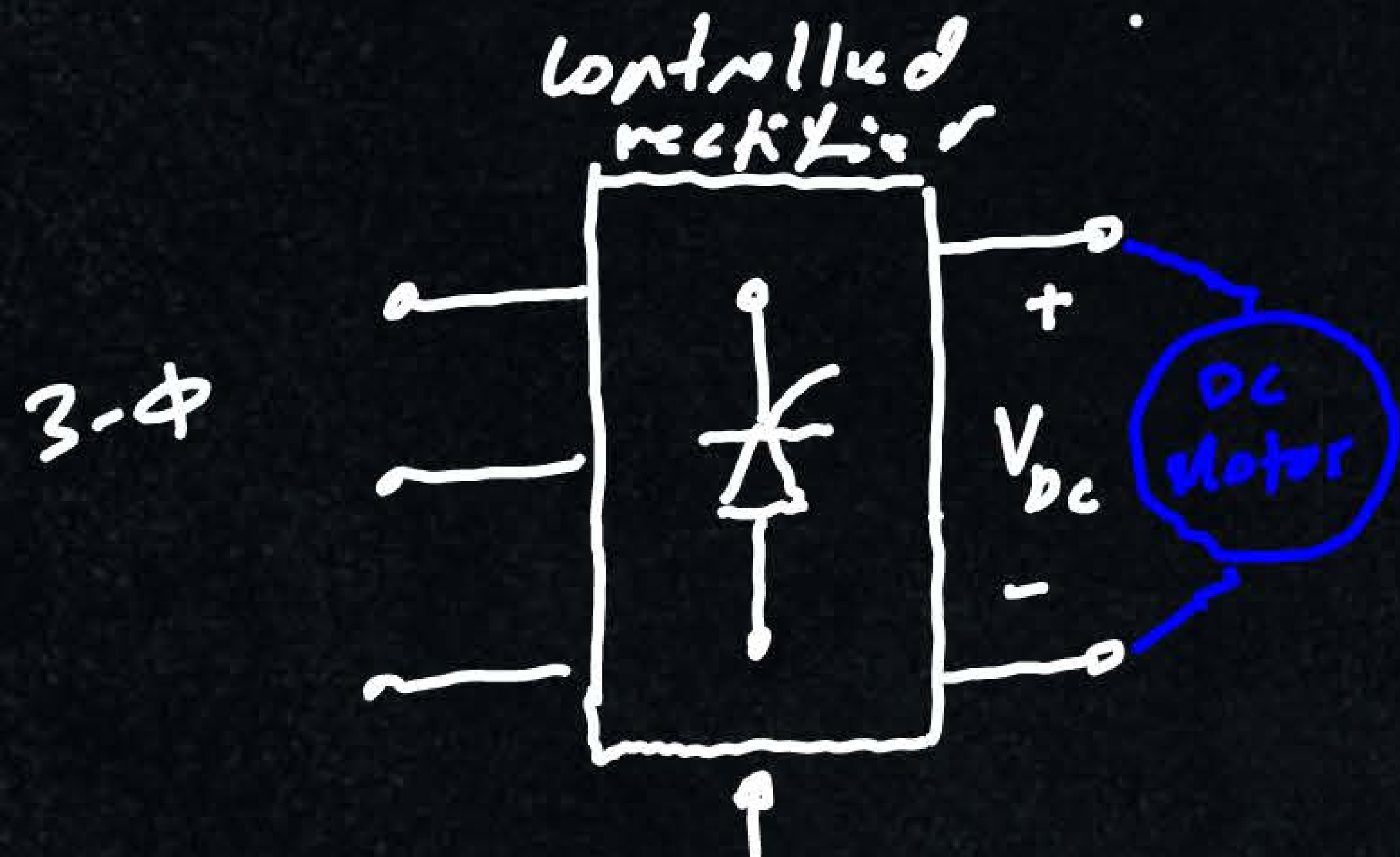
$$0 \le M \le 1$$

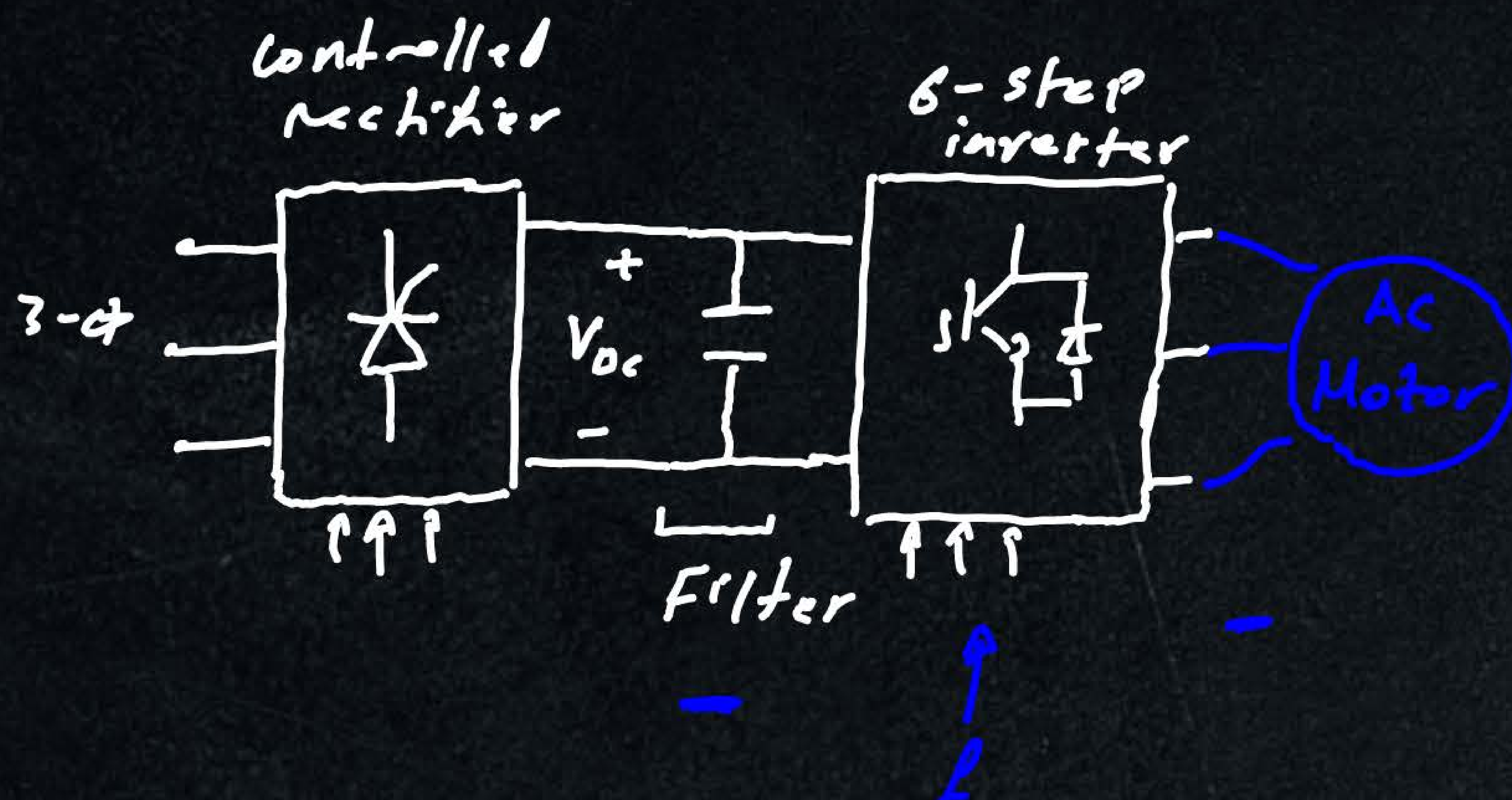
$$0 \le V_{L-N,\,peak} \le \frac{V_s}{2}$$

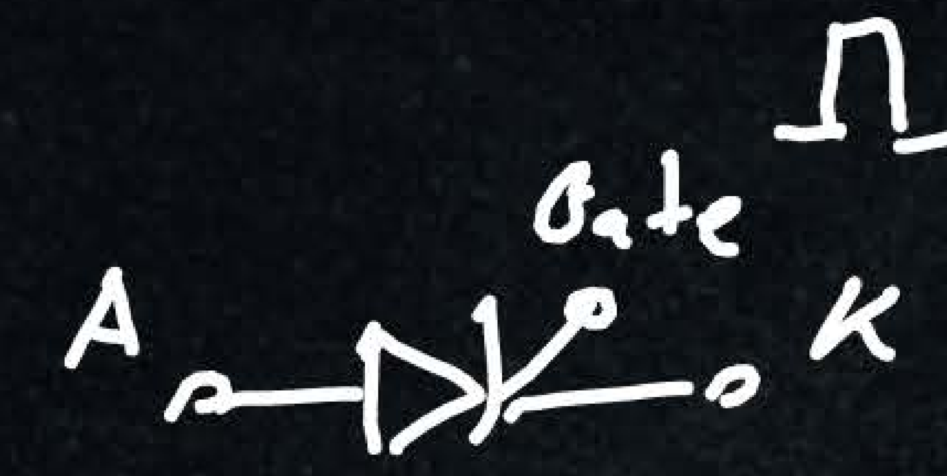
1	3	5
4	6	2



Controlled Rectifiers

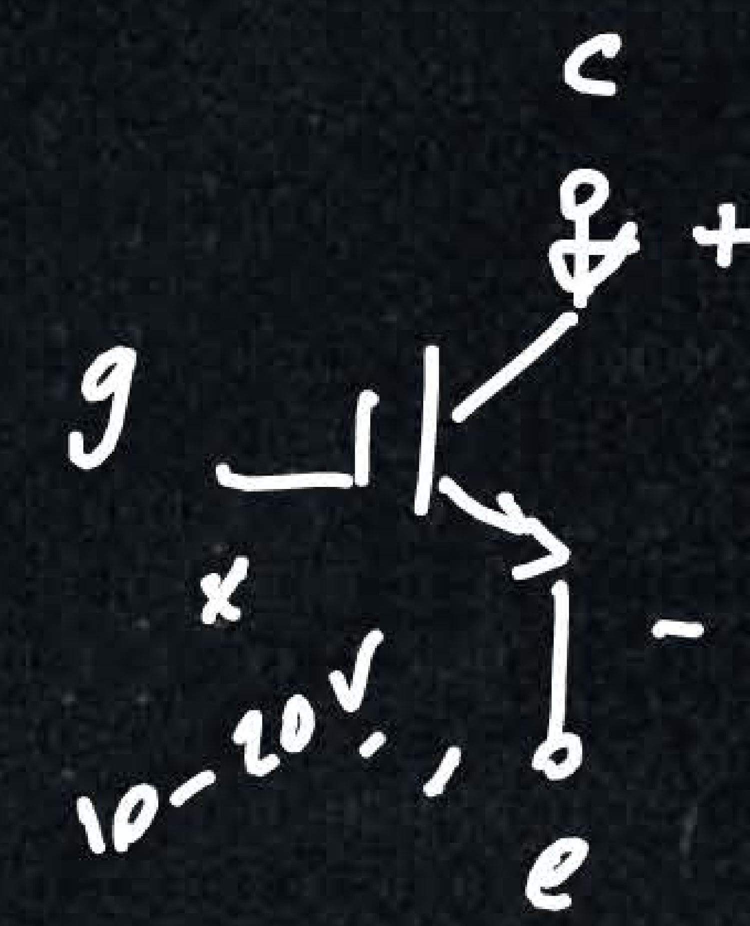




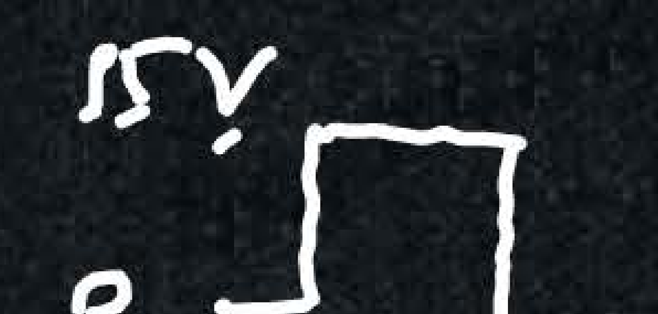


SCR

Thyristor

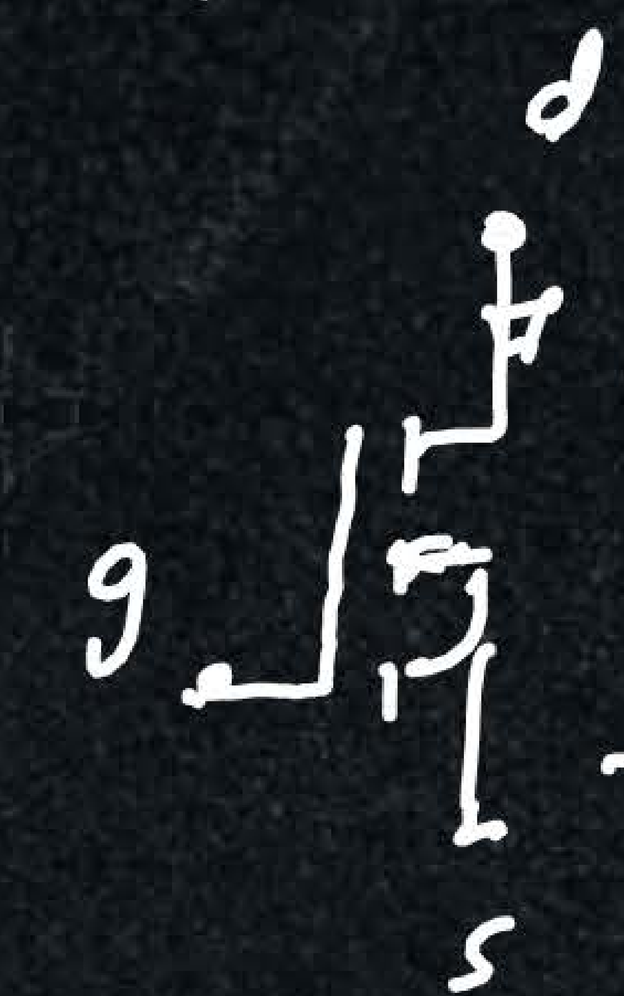


IGBT



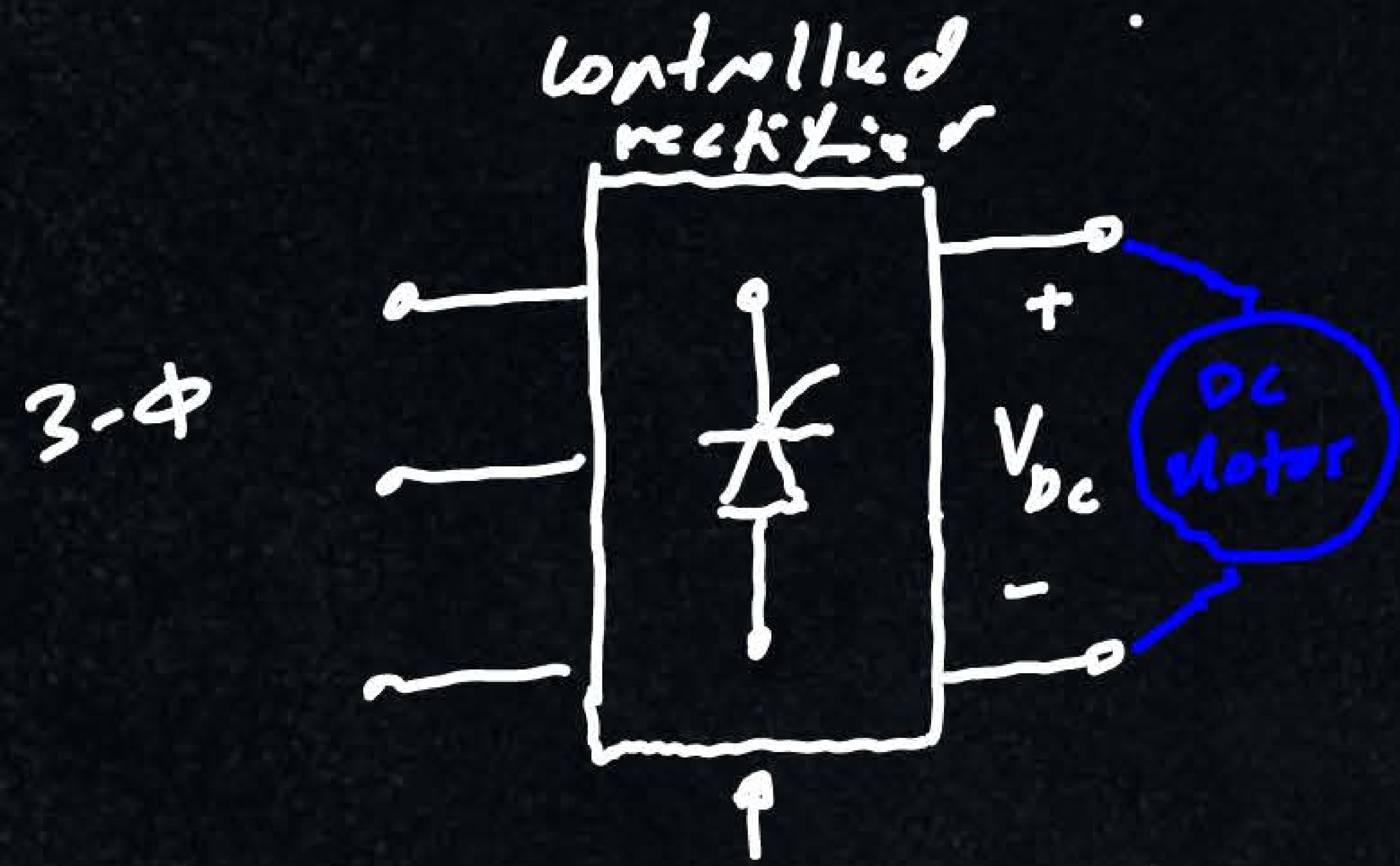
ON OFF

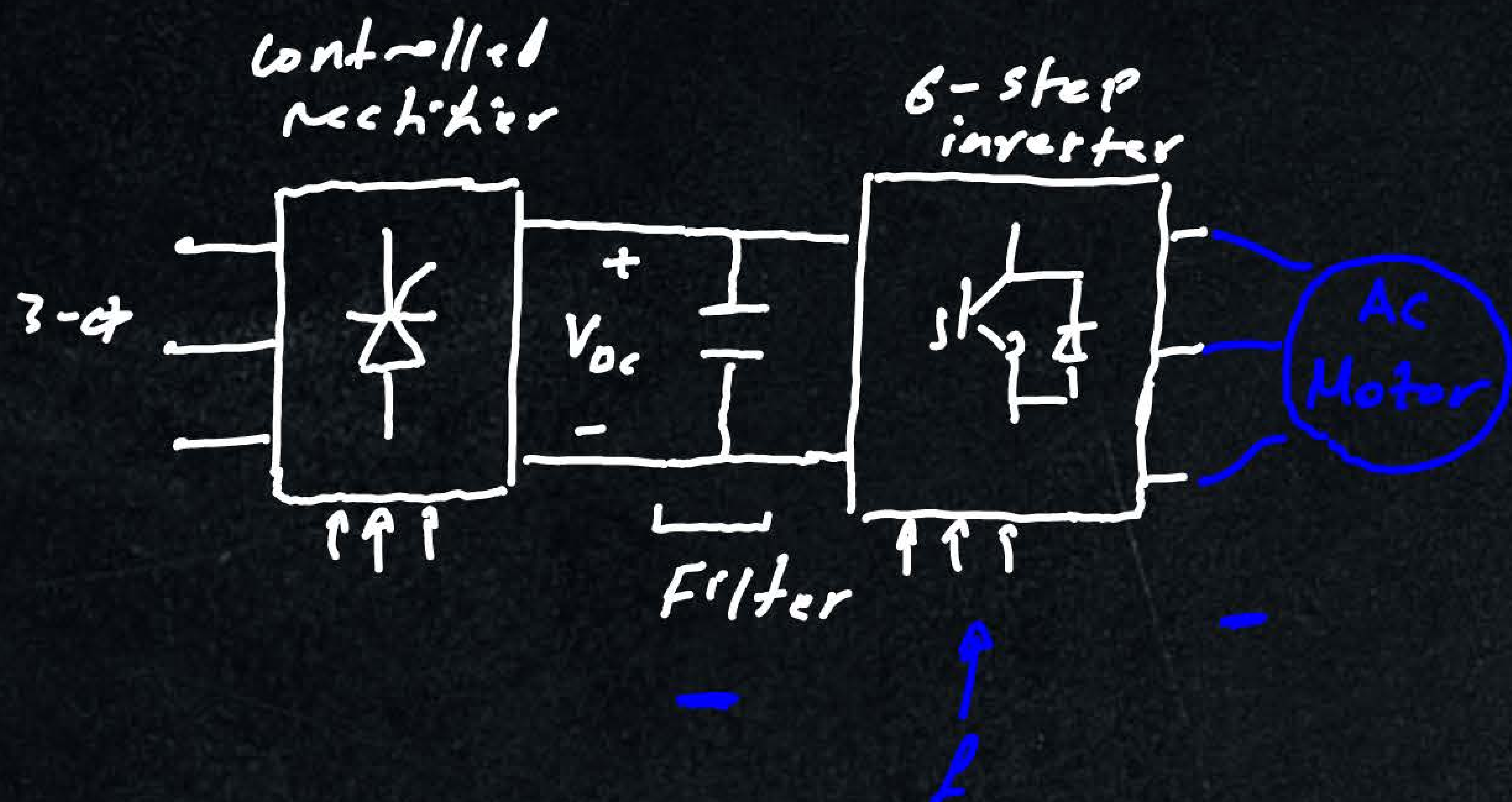
short open

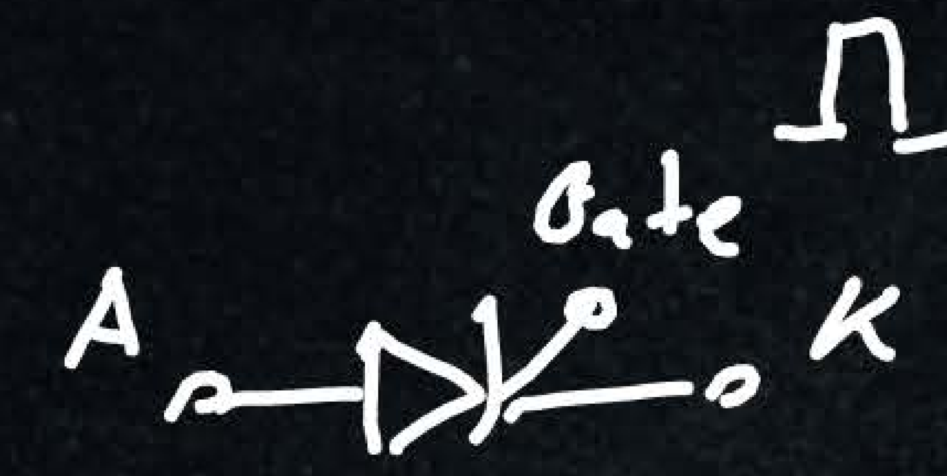


100 kHz

Controlled Rectifiers

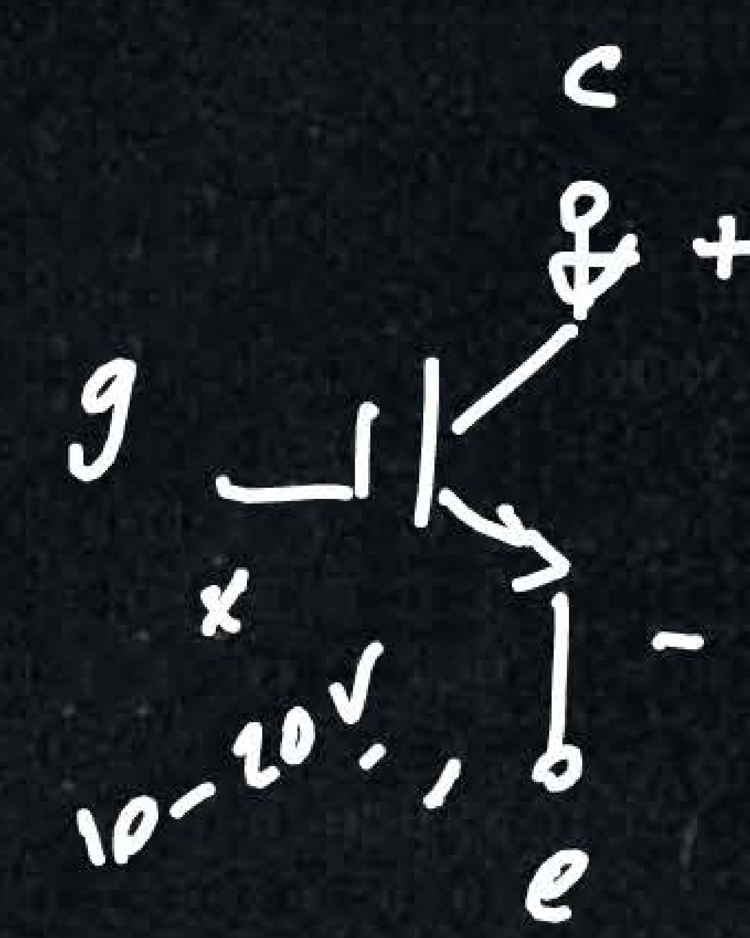




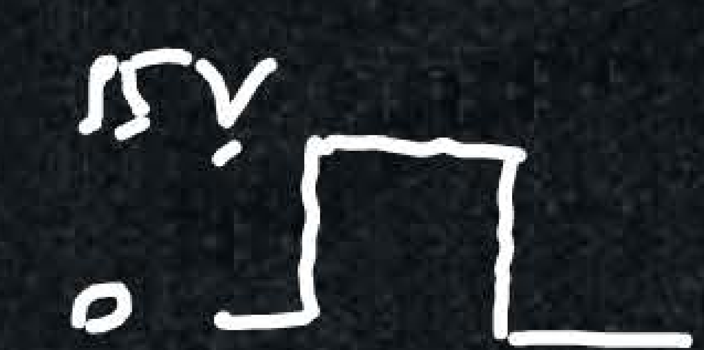


SCR

Thyristor

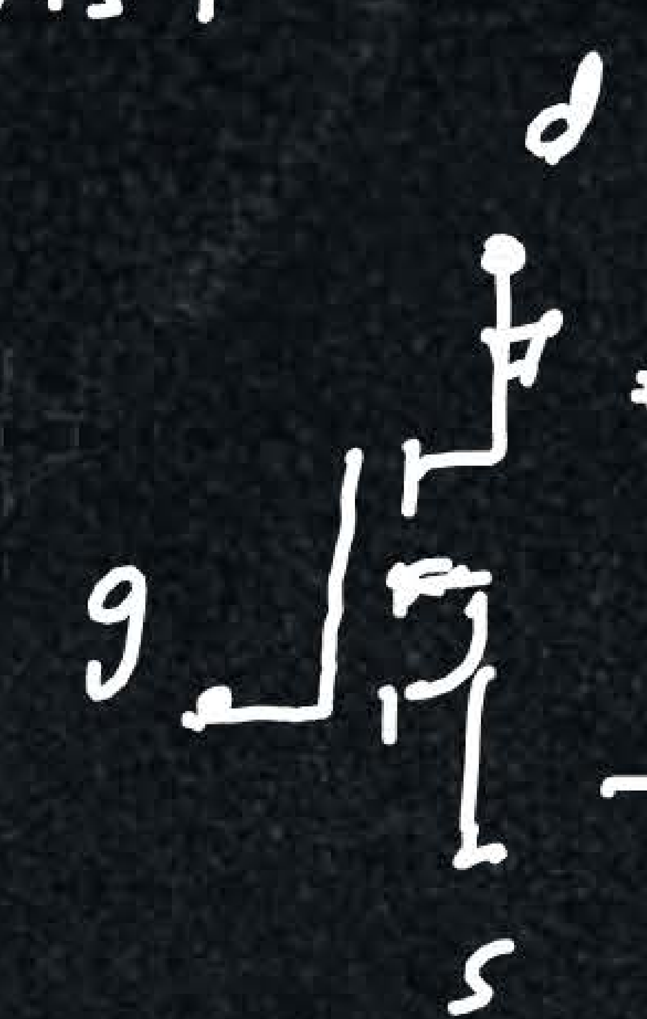


IGBT



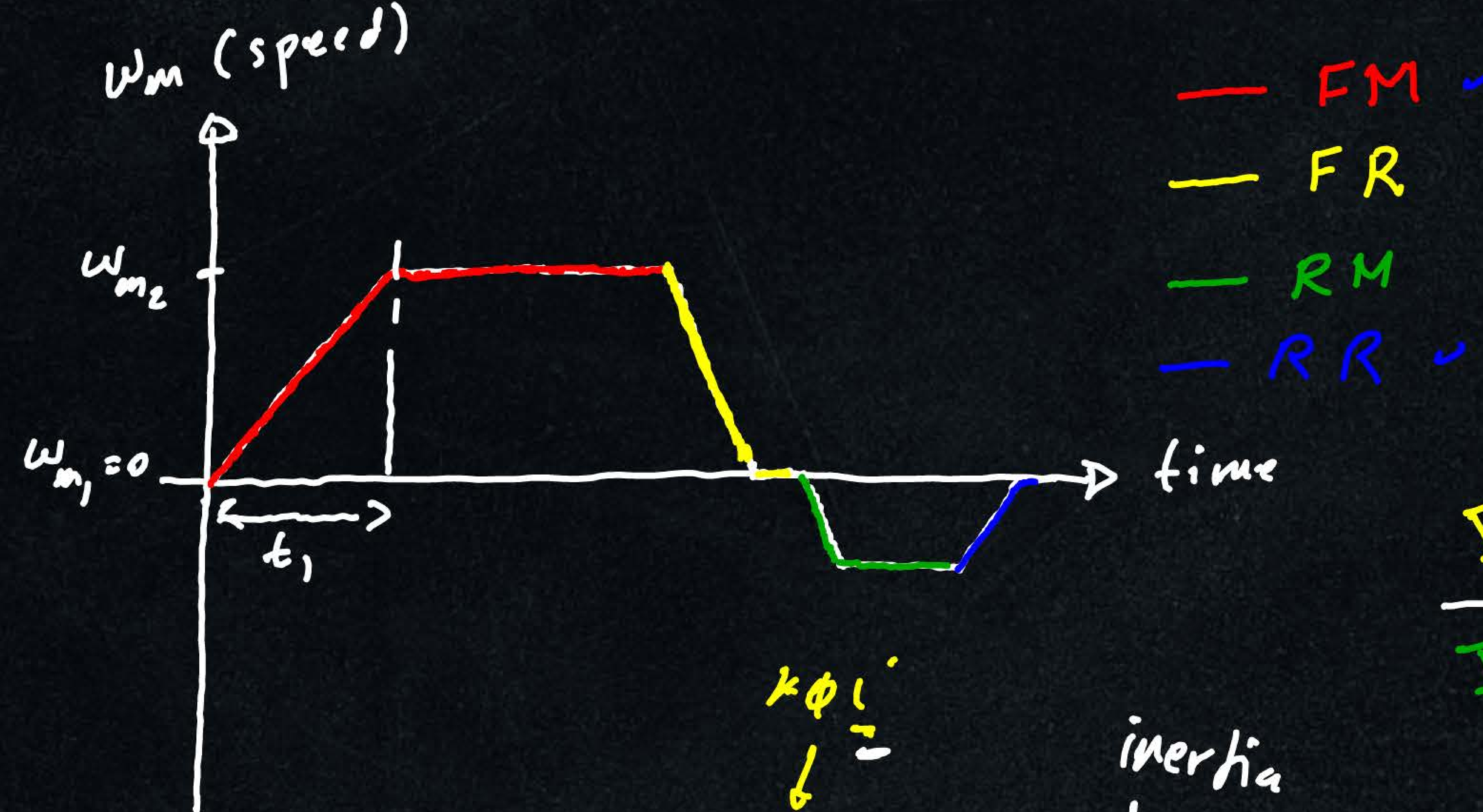
ON OFF

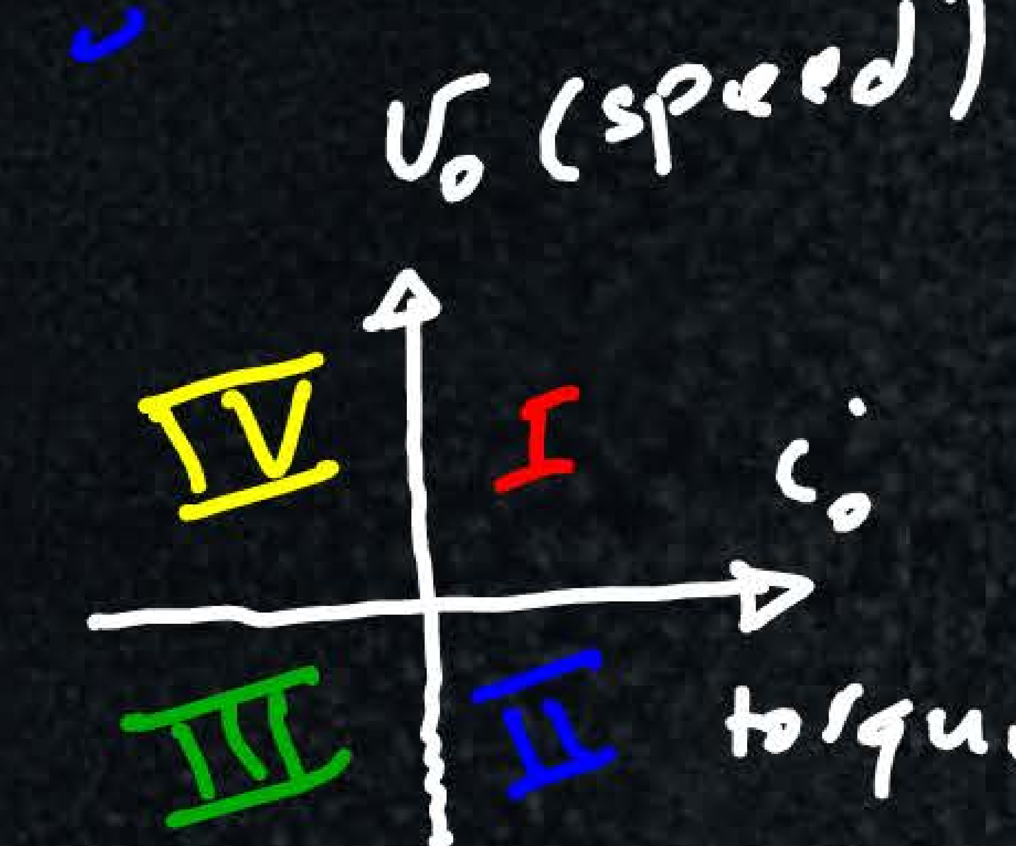
short open



100 kHz

MOSFET





$kφi$

inertia

$$T_{el} - T_l = J\frac{d\omega_m}{dt}$$

electro-magnet torque

load torque

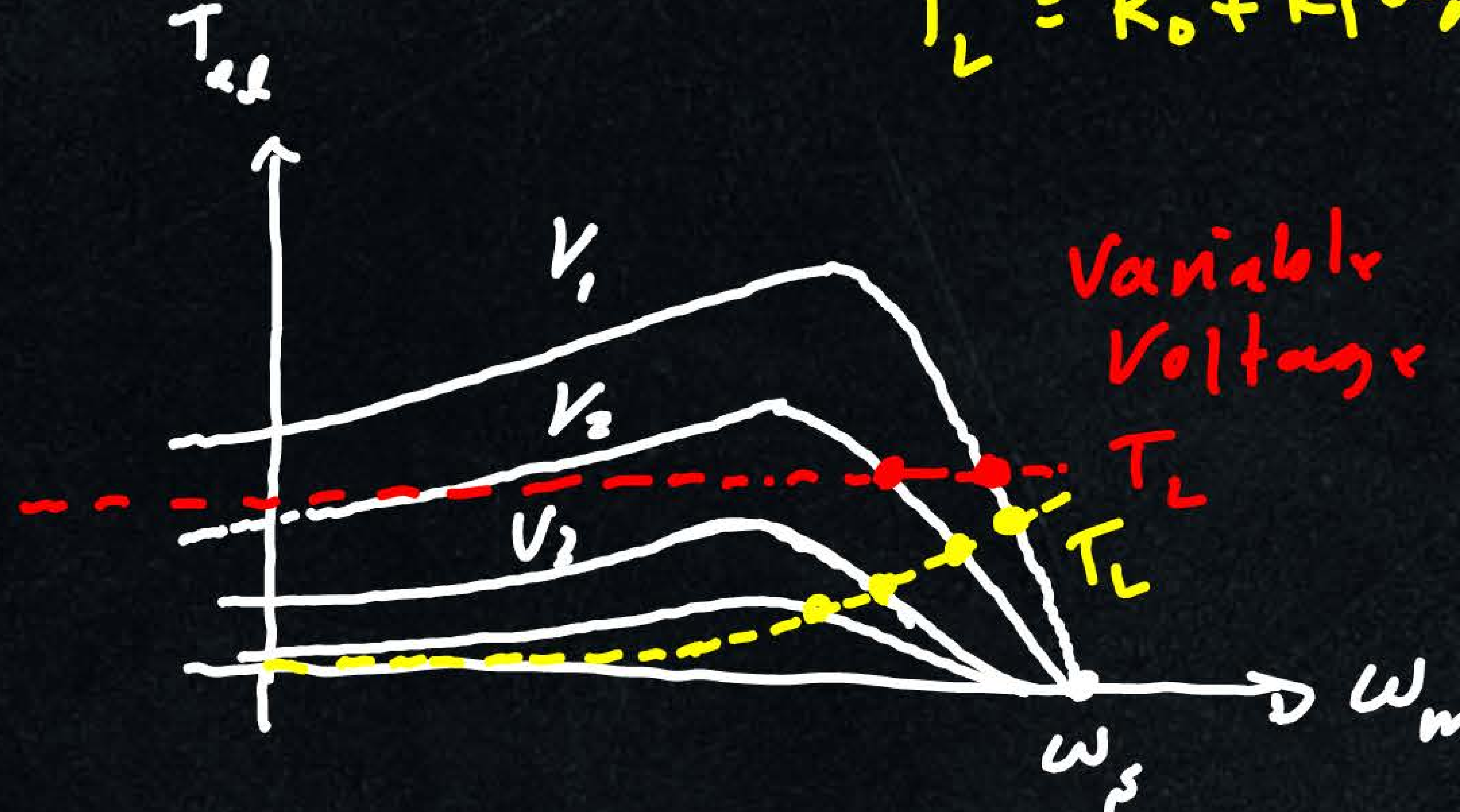
acceleration

$$T_{el} - T_l = J\frac{\omega_{m_2}}{t_1}$$

$$t_1 = J\frac{\omega_{m_2}}{\Sigma T}$$

$T_L = k_0 + k_1 \omega_m + k_2 \omega_m^2 + \dots$

$\omega_s = \frac{\omega_e}{p} = \frac{2\pi f_e}{p}$



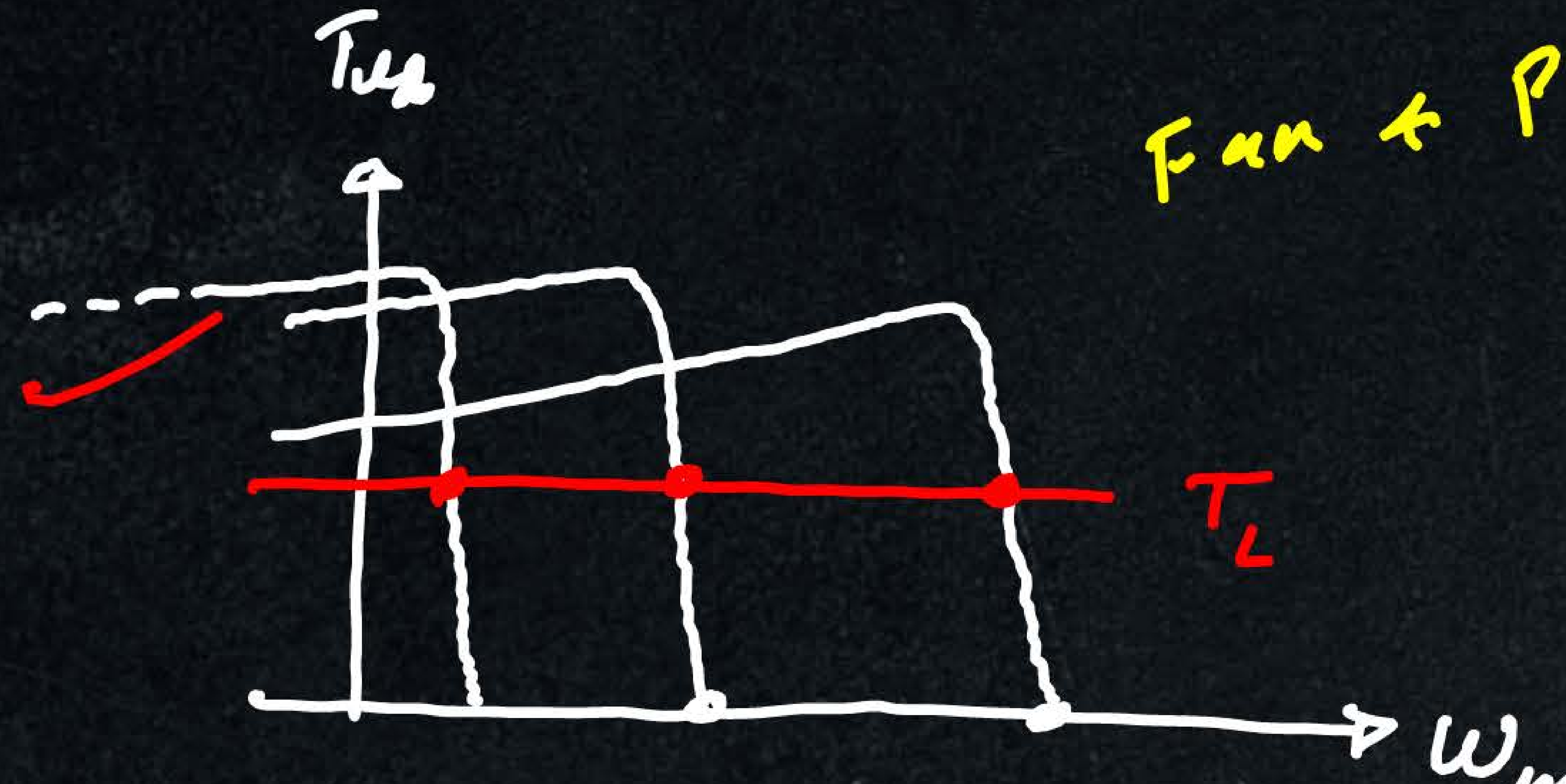
$V_2 < V_1$

V_1 = rated voltage

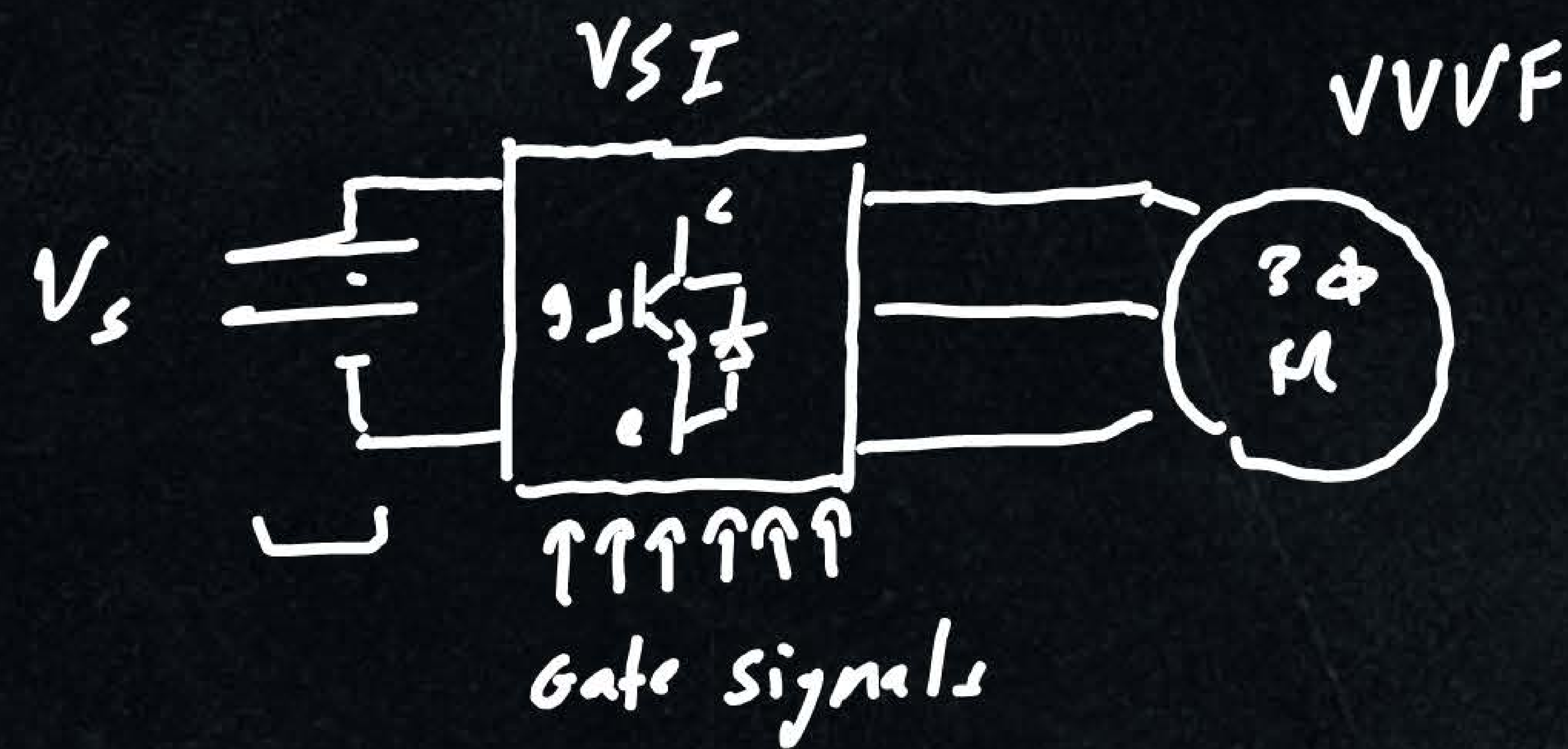
$T_L \propto \omega_m^2$

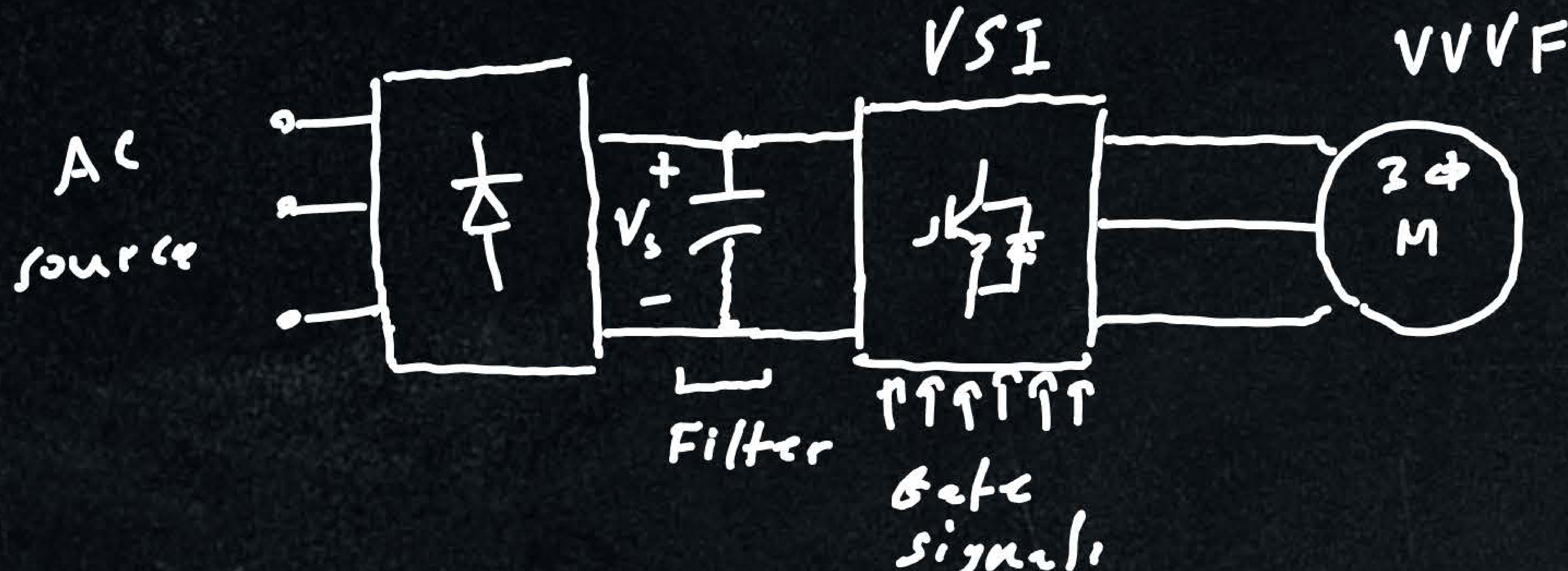
Fan & pump drives

$T_{el} = T_L + J \frac{d\omega_m}{dt}$



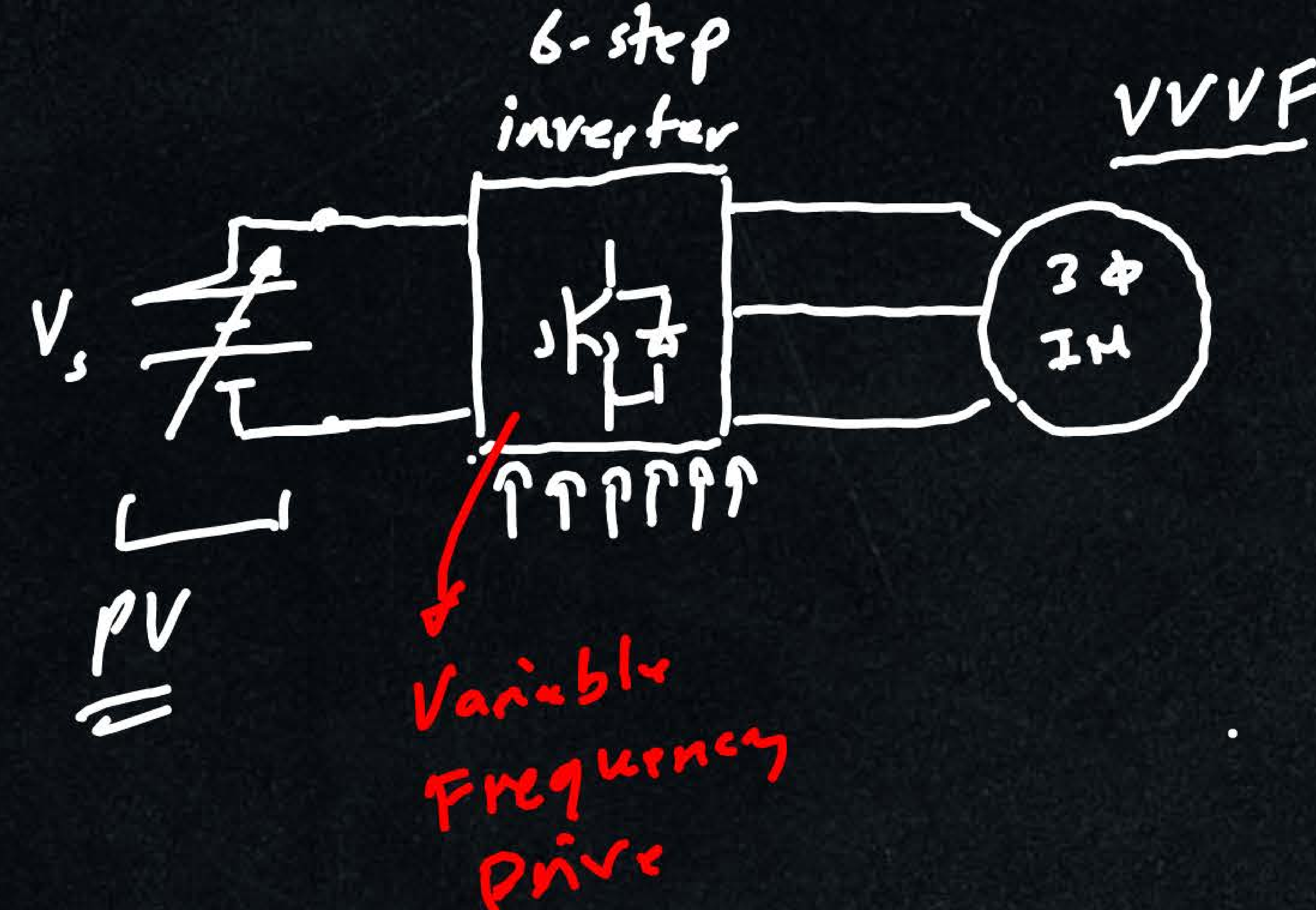
Variable voltage variable frequency.





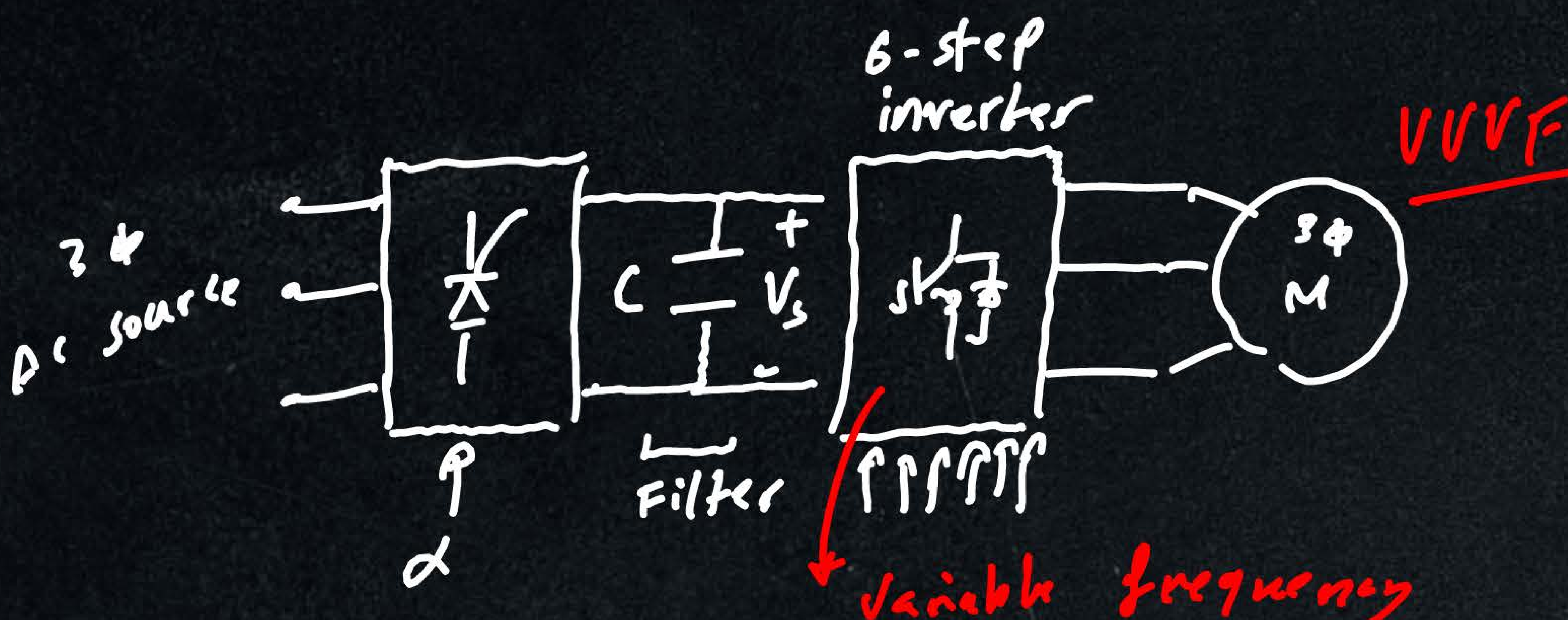
V_s = constant

(VSI)

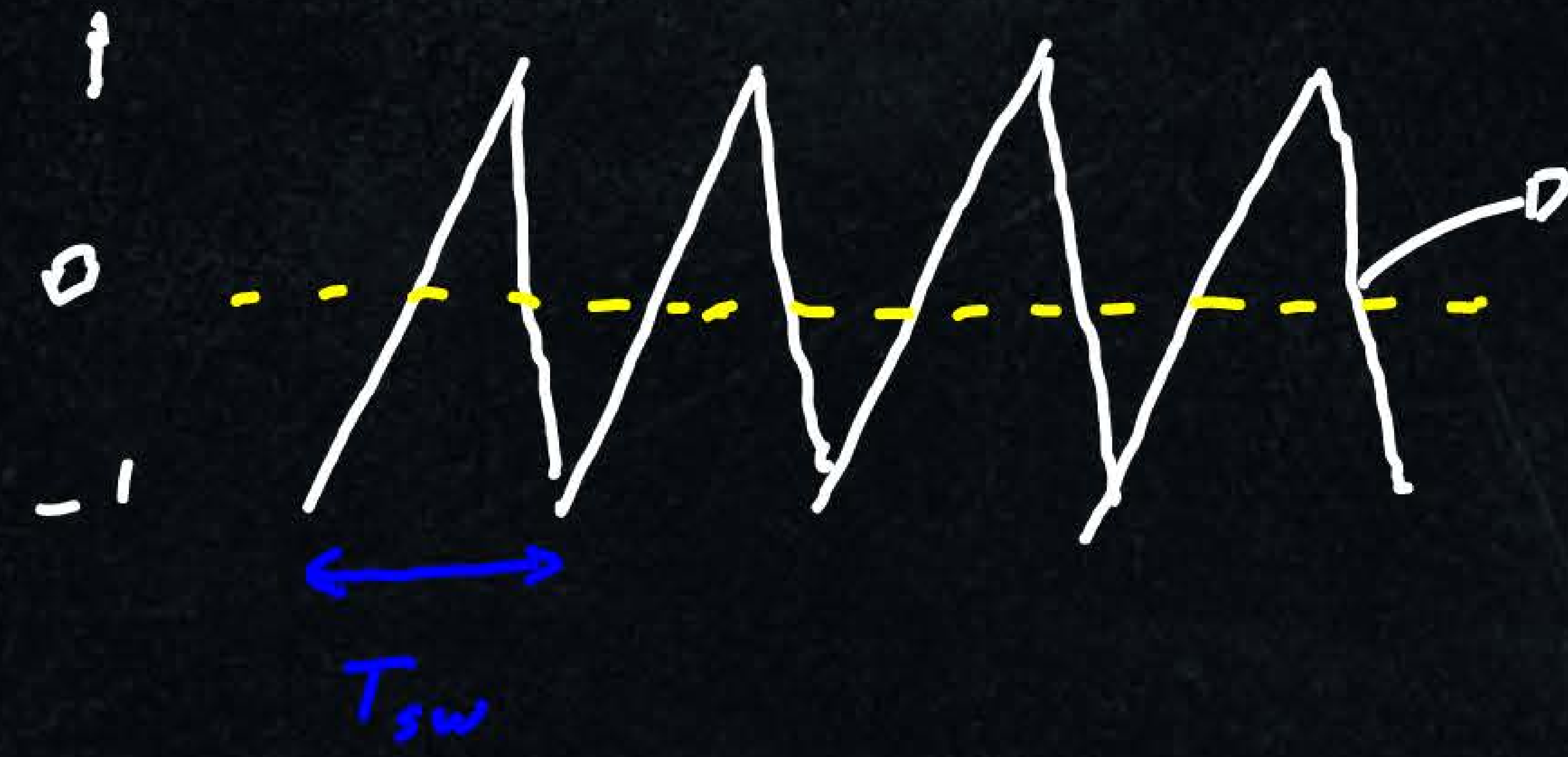


V_s is variable DC voltage

(6-step inverter)



SPWM



carrier signal

" $v_{carrier}$ "

switching period

$f_{sw} = \frac{1}{T_{sw}} \equiv$ switching frequency

carrier frequency

$f_{sw} \gg f_1$

f_1: Fundamental Frequency of output voltage three modulating waves

400V L-L rms

326V L-N peak

$$m_a = M \sin(\omega_e t)$$

$$m_b = M \sin(\omega_e t - 2\pi/3)$$

$$m_c = M \sin(\omega_e t + 2\pi/3)$$

ω_e : Electric radian frequency

$\omega_e = 2\pi f_{e,1}$

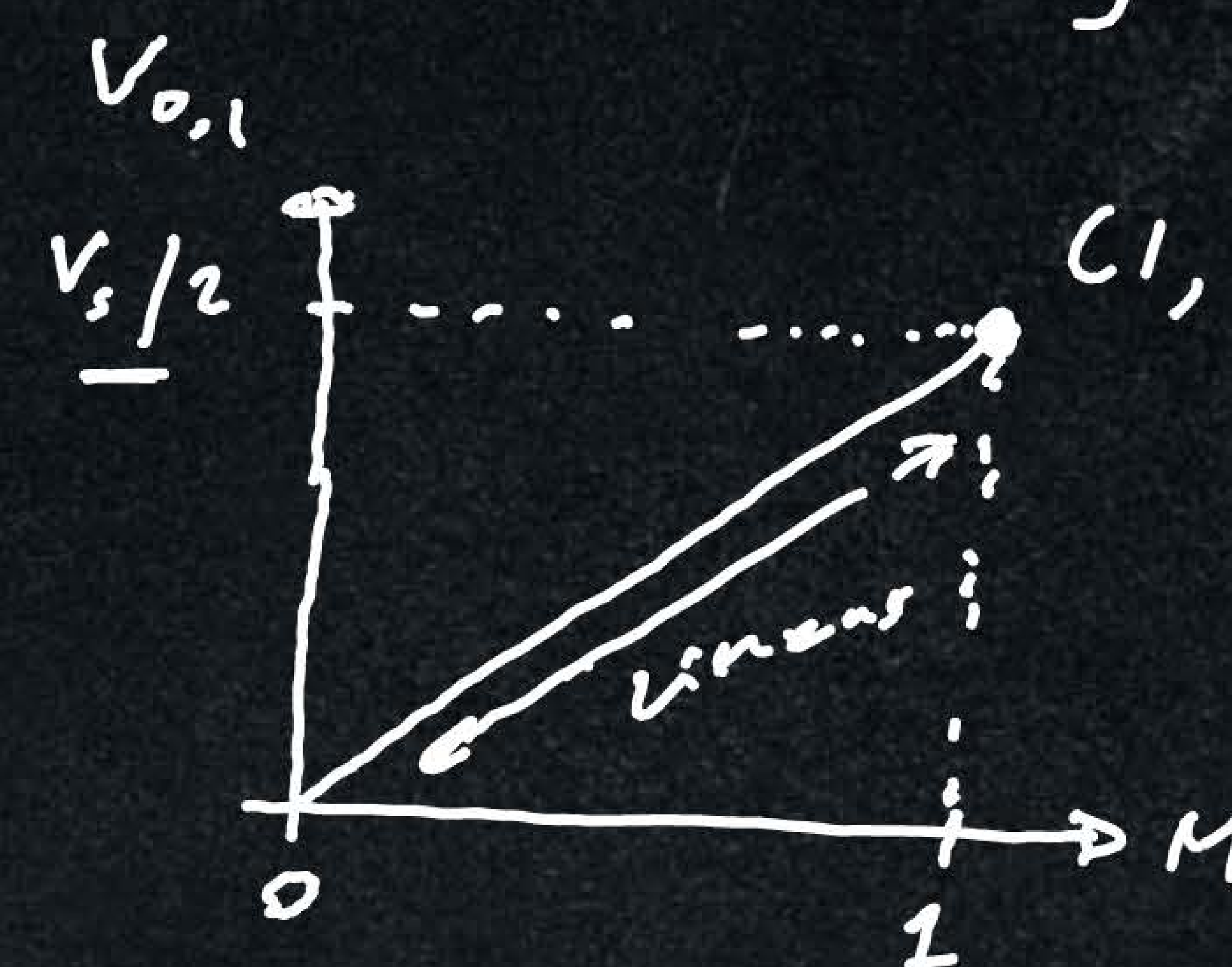
$f_{e,1}$ ↓ Fundamental electric frequency

$$0 \le M \le 1$$

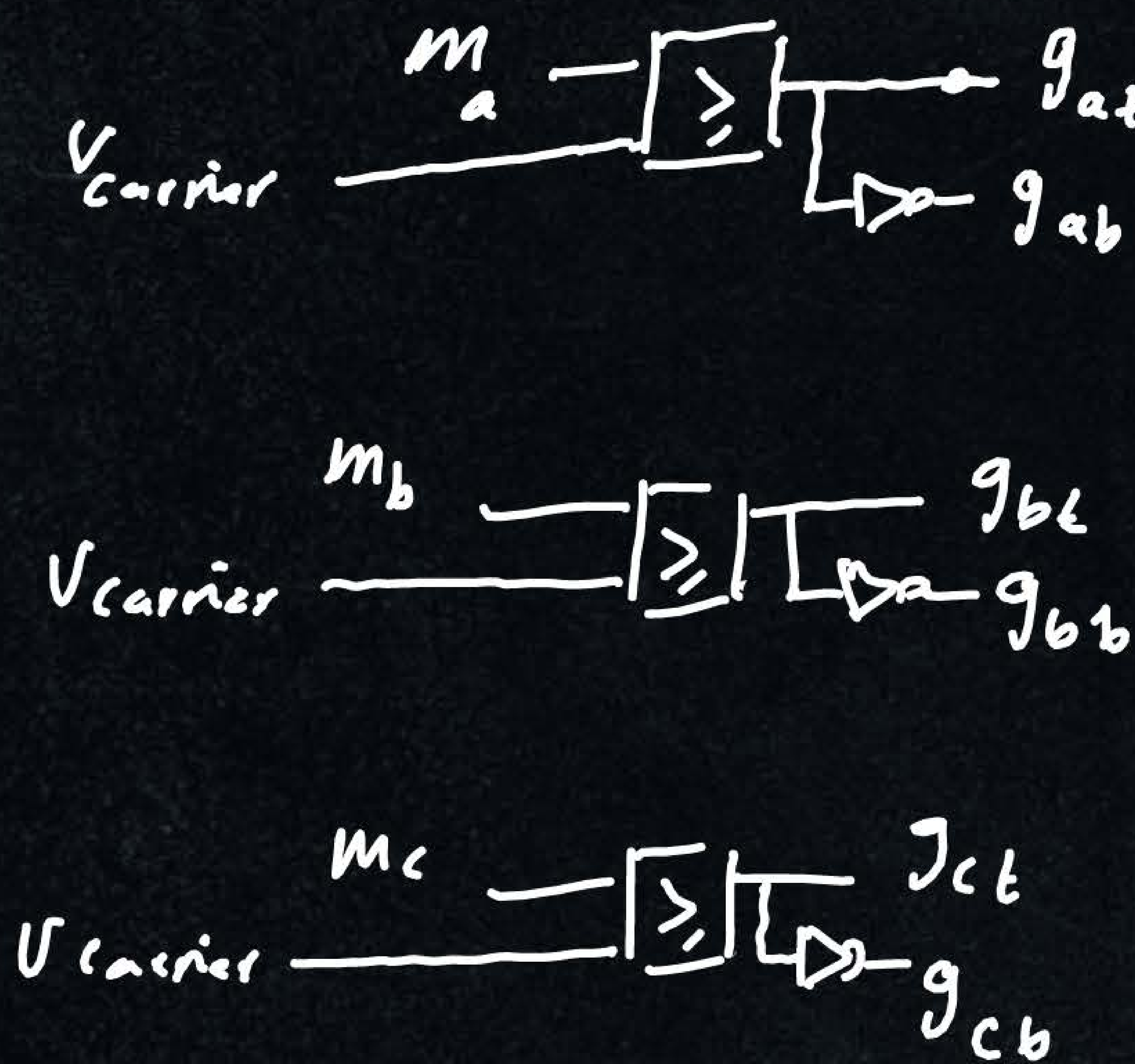
$$V_{o,1} = M \frac{V_s}{2}$$ L-N, peak voltage

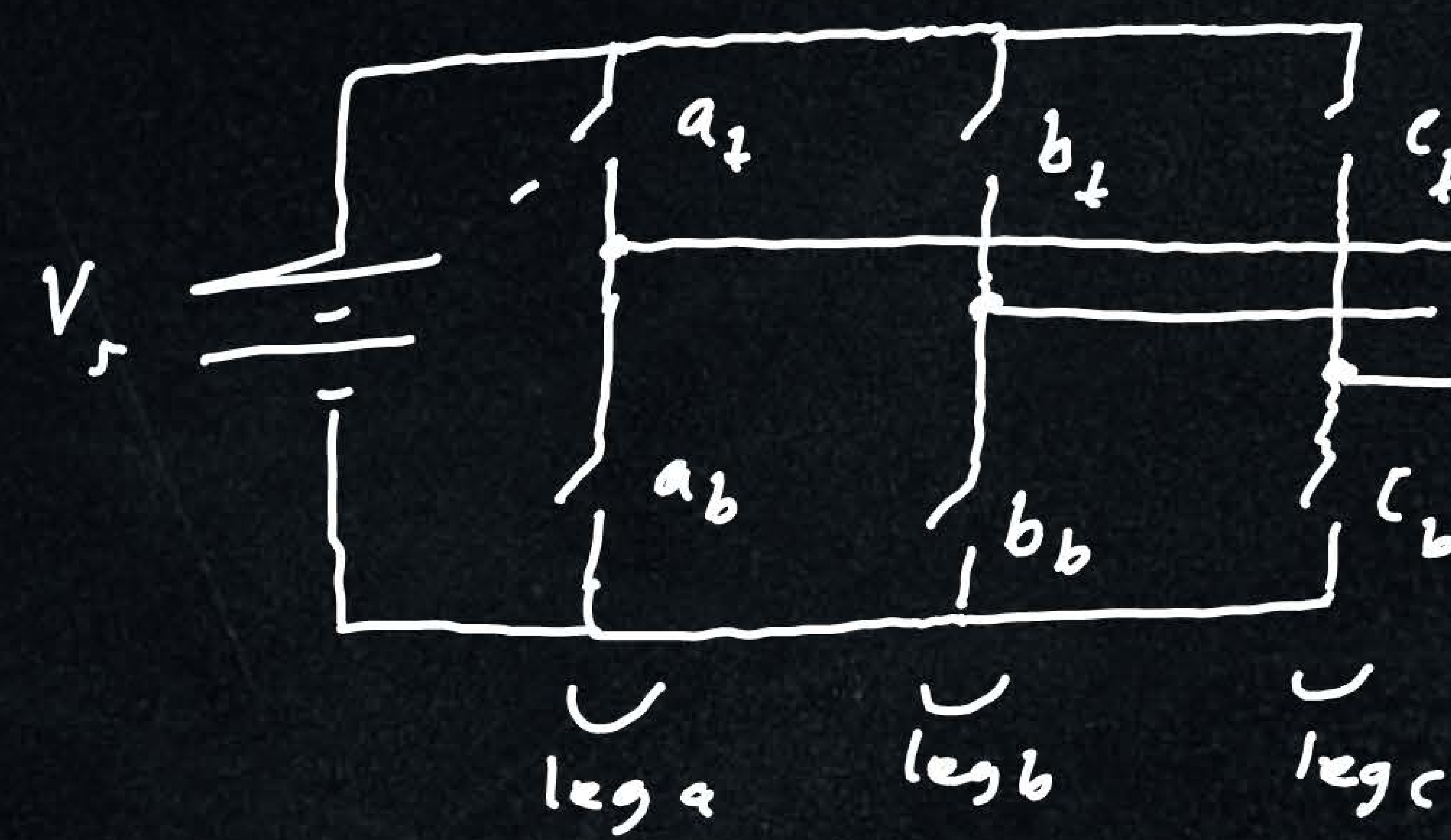
$$0 \le V_{o,1} \le \frac{V_s}{2}$$

400V L-L rms ⟶ 653V = V_s



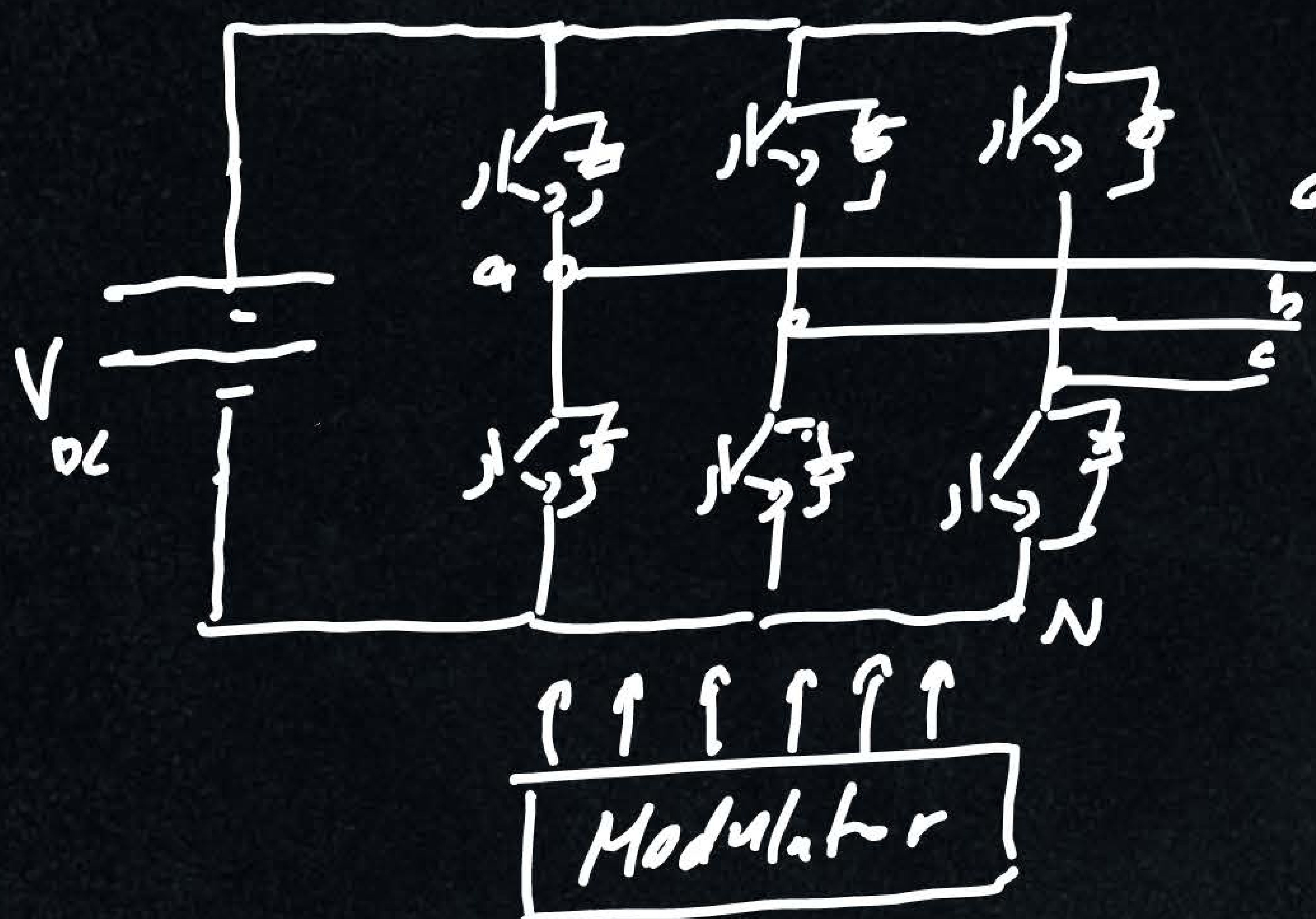
$V_{o,1}$: peak phase voltage (main comp.)


m_a
$V_{carrier}$
$\geq$
g_{at}
g_{ab}
m_b
$V_{carrier}$
$\geq$
g_{bt}
g_{bb}
m_c
$V_{carrier}$
$\geq$
g_{ct}
g_{cb}

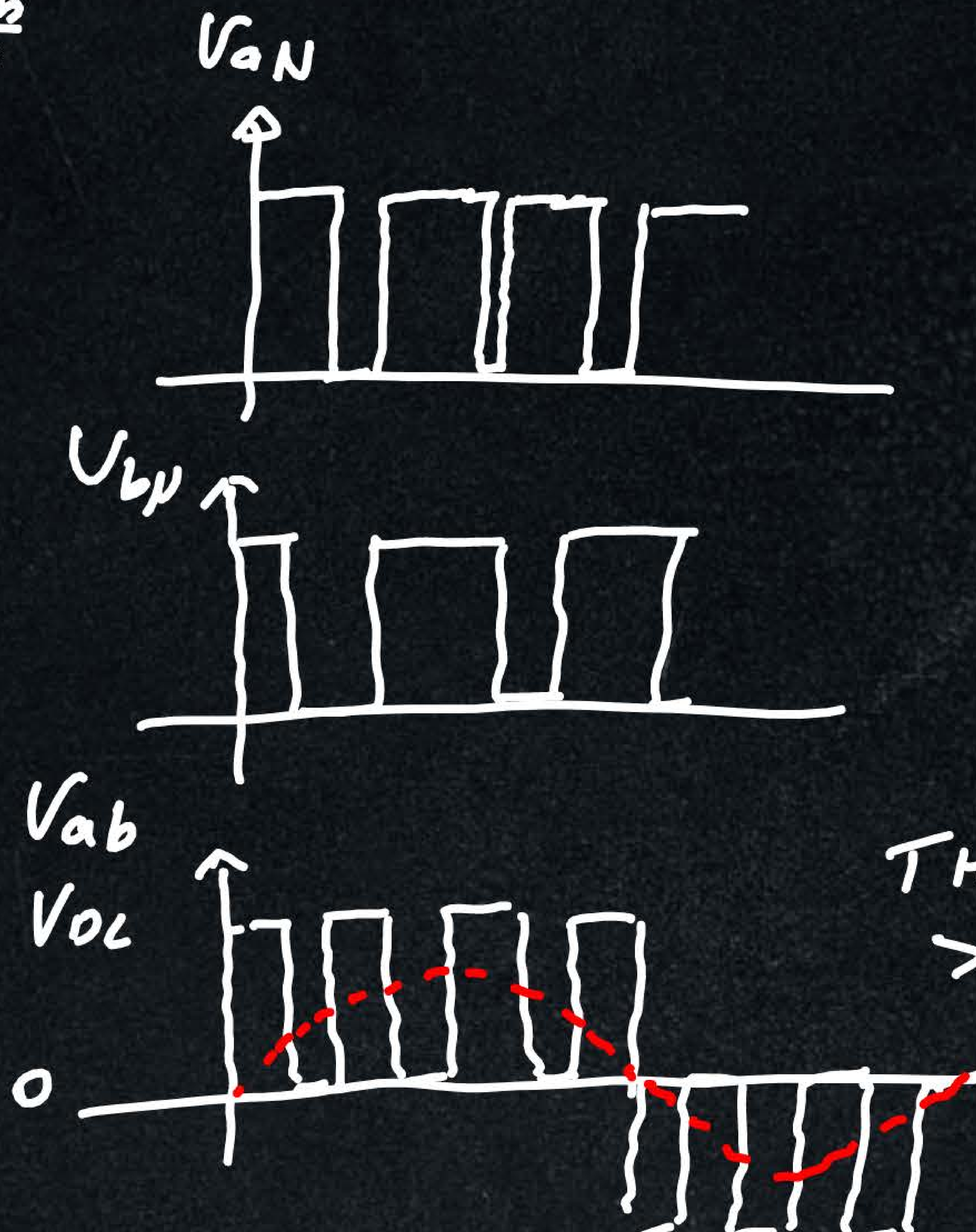

V_s
a_t
b_t
c_t
a_b
b_b
c_b
leg a
leg b
leg c

Voltage source inverter

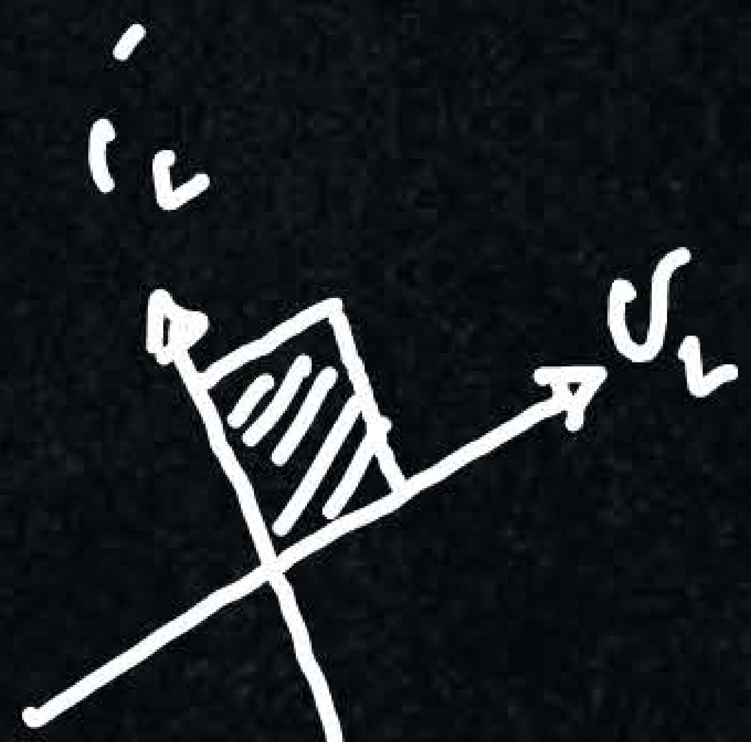
(Traditional inverter)



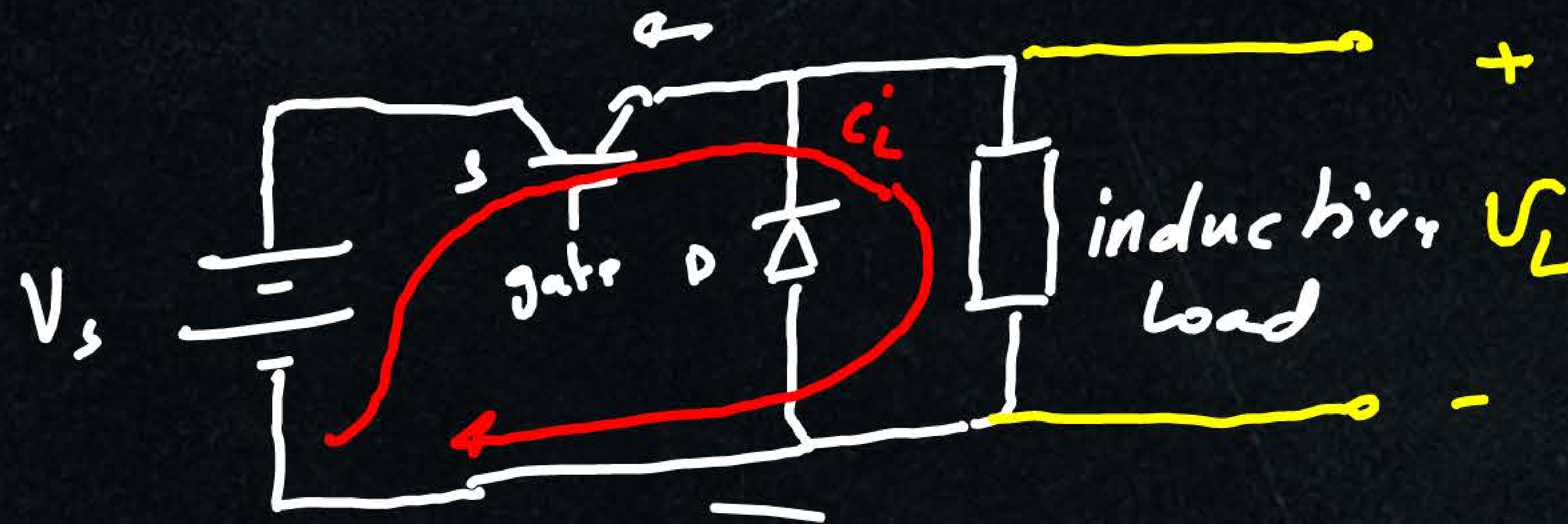
2-level inverter



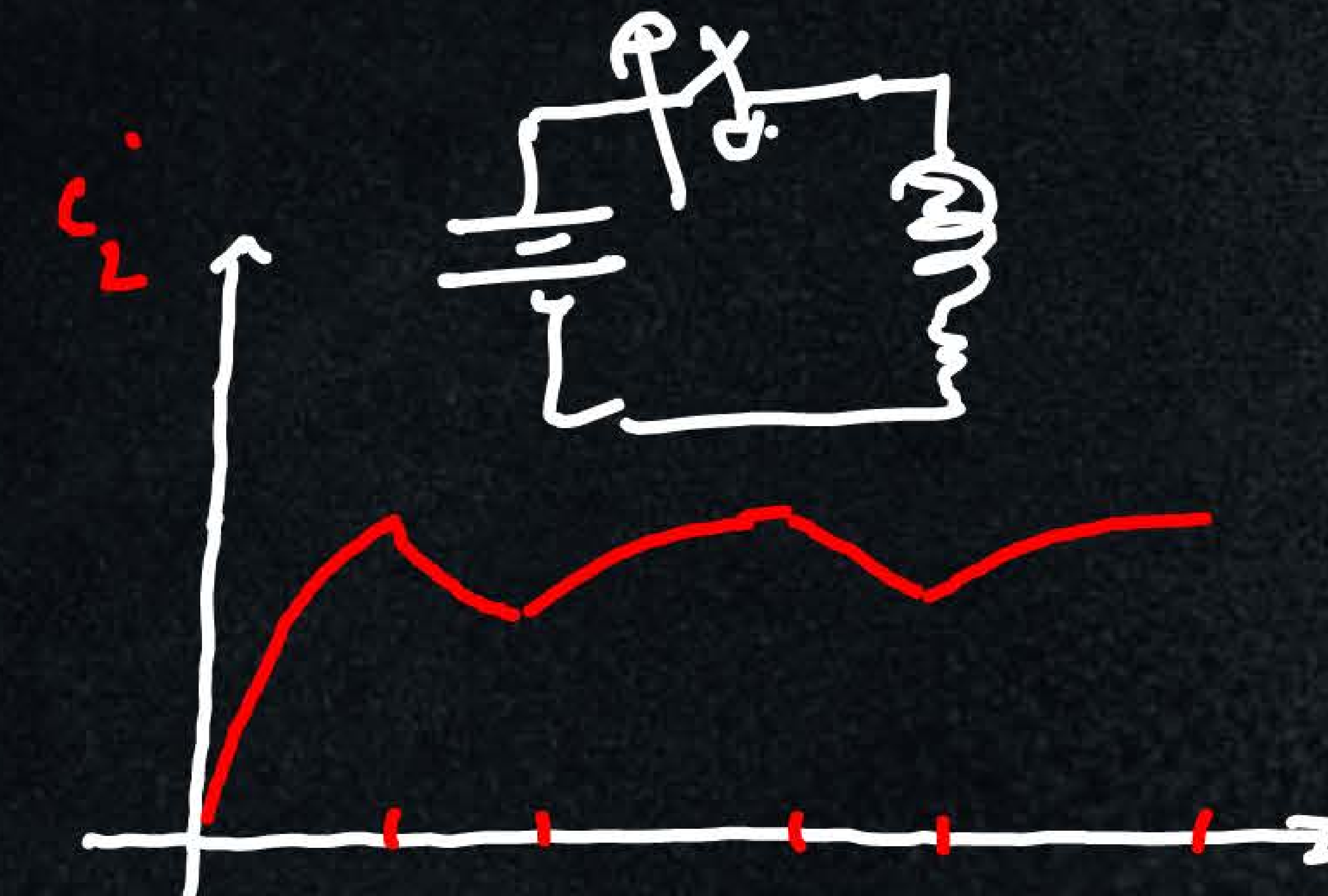
THD >70%

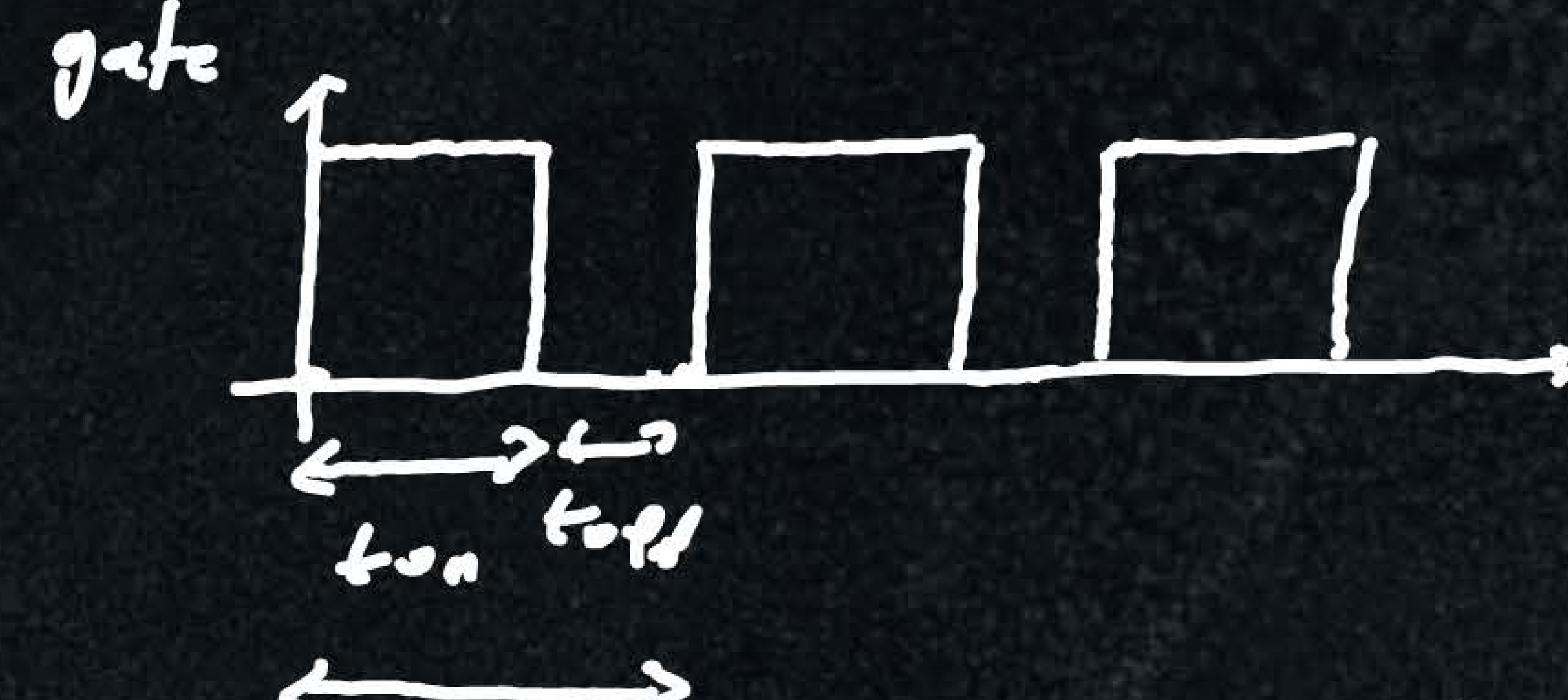


S is ON

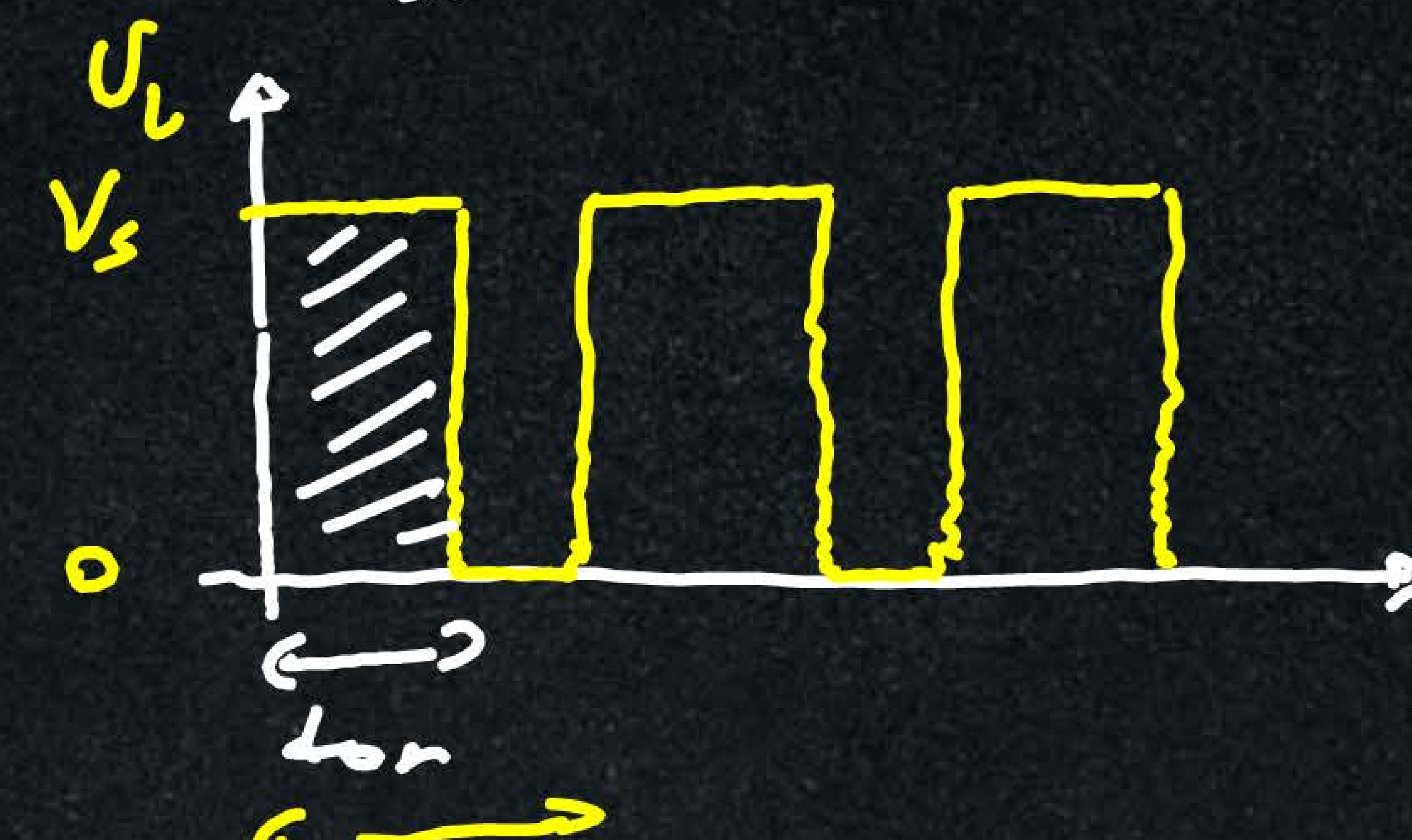


S is OFF





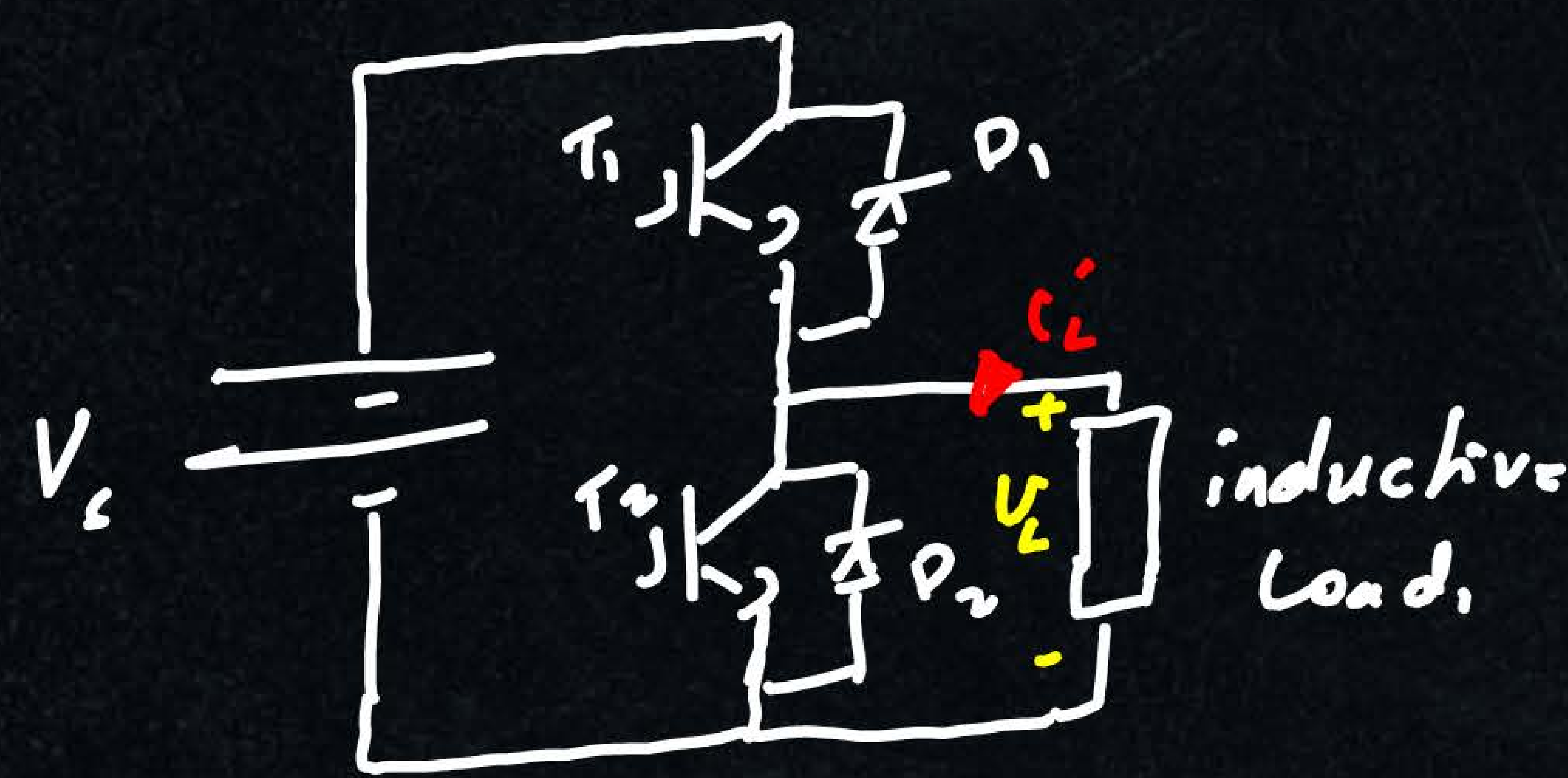
t_{sw} : switching period



The average output voltage, V_L, across the load is calculated as:

$$V_L = \frac{1}{t_{sw}} \int_0^{t_{sw}} v_L(t)\,dt = \frac{t_{on}}{t_{sw}} V_s$$

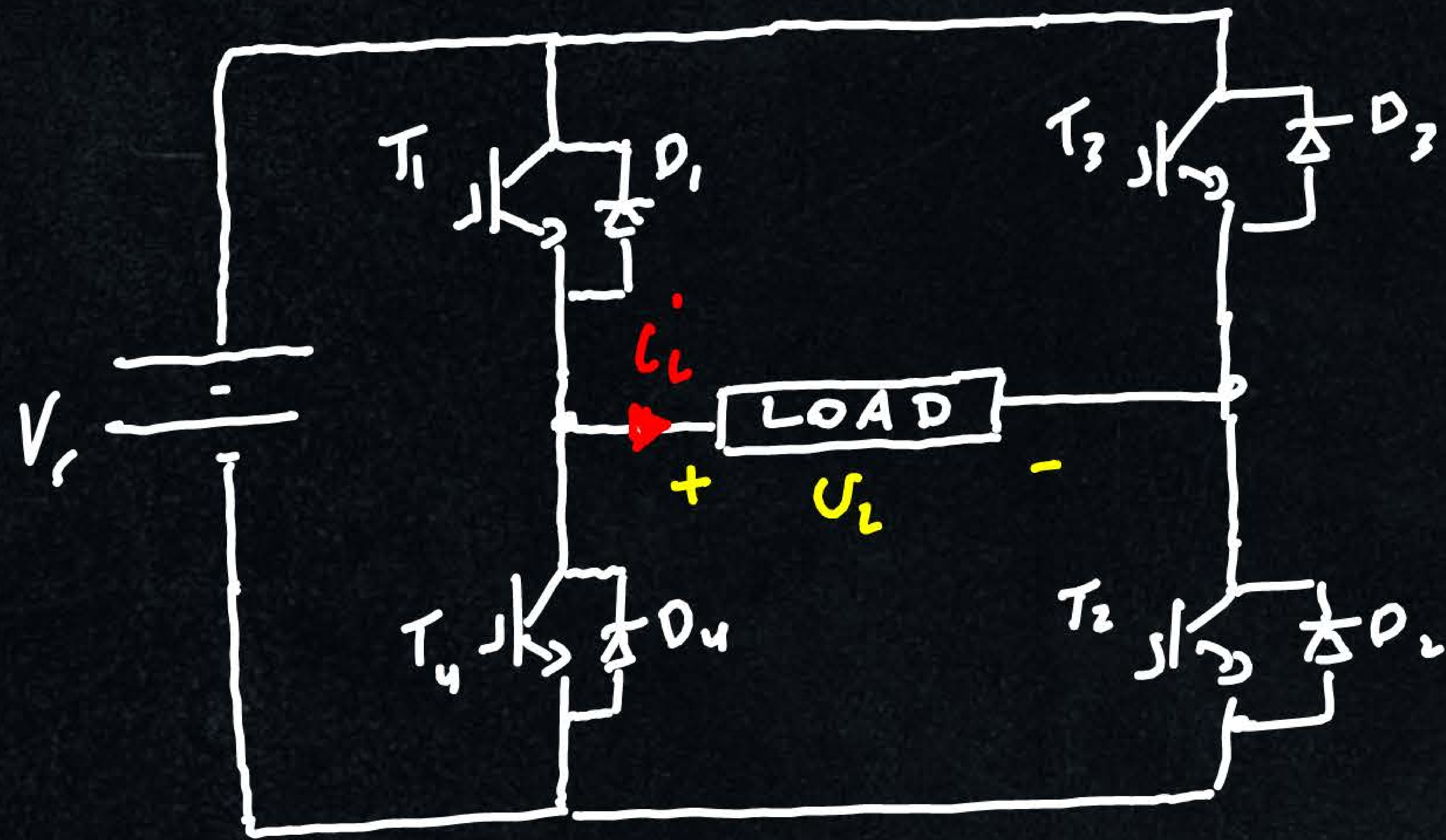
$V_L = k V_s$ where k is the duty cycle $0 \leq k \leq 1$

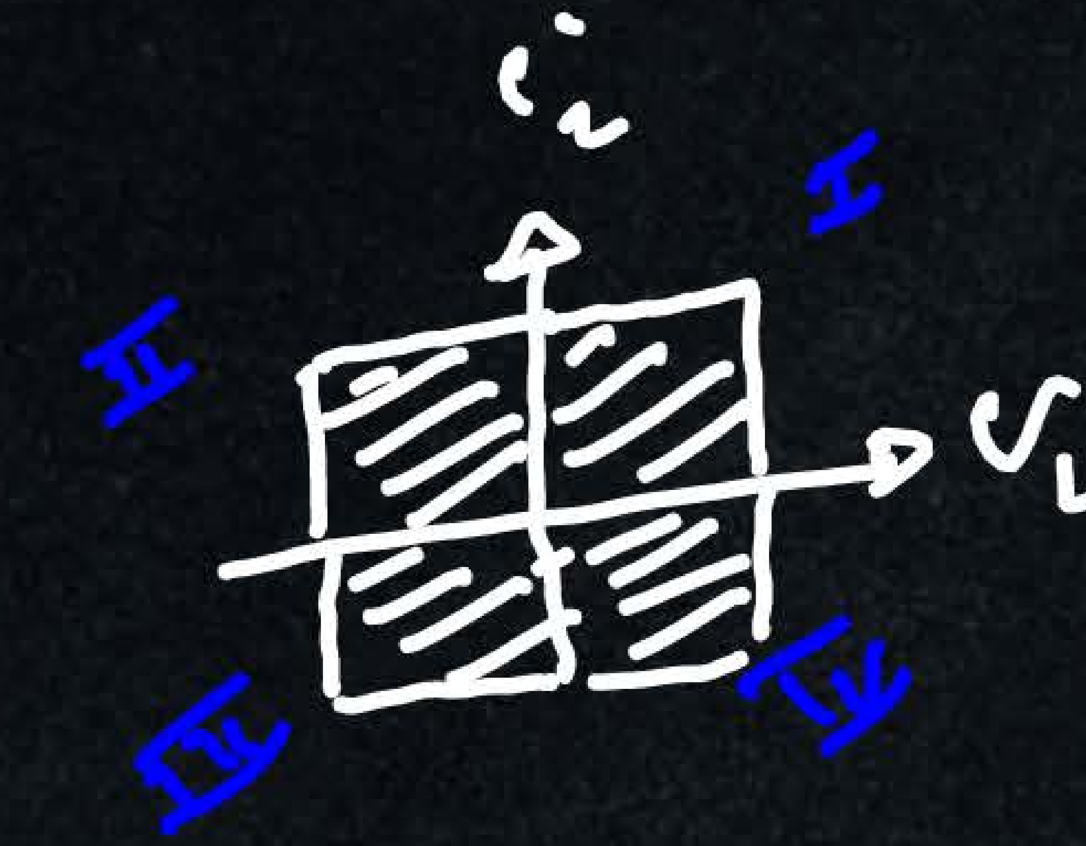


Half-bridge chopper circuit

Two-quadrant chopper circuit

v_L	i_L	conducting device.
V_s	+	T_1
0	+	D_2
V_s	−	D_1
0	−	T_2





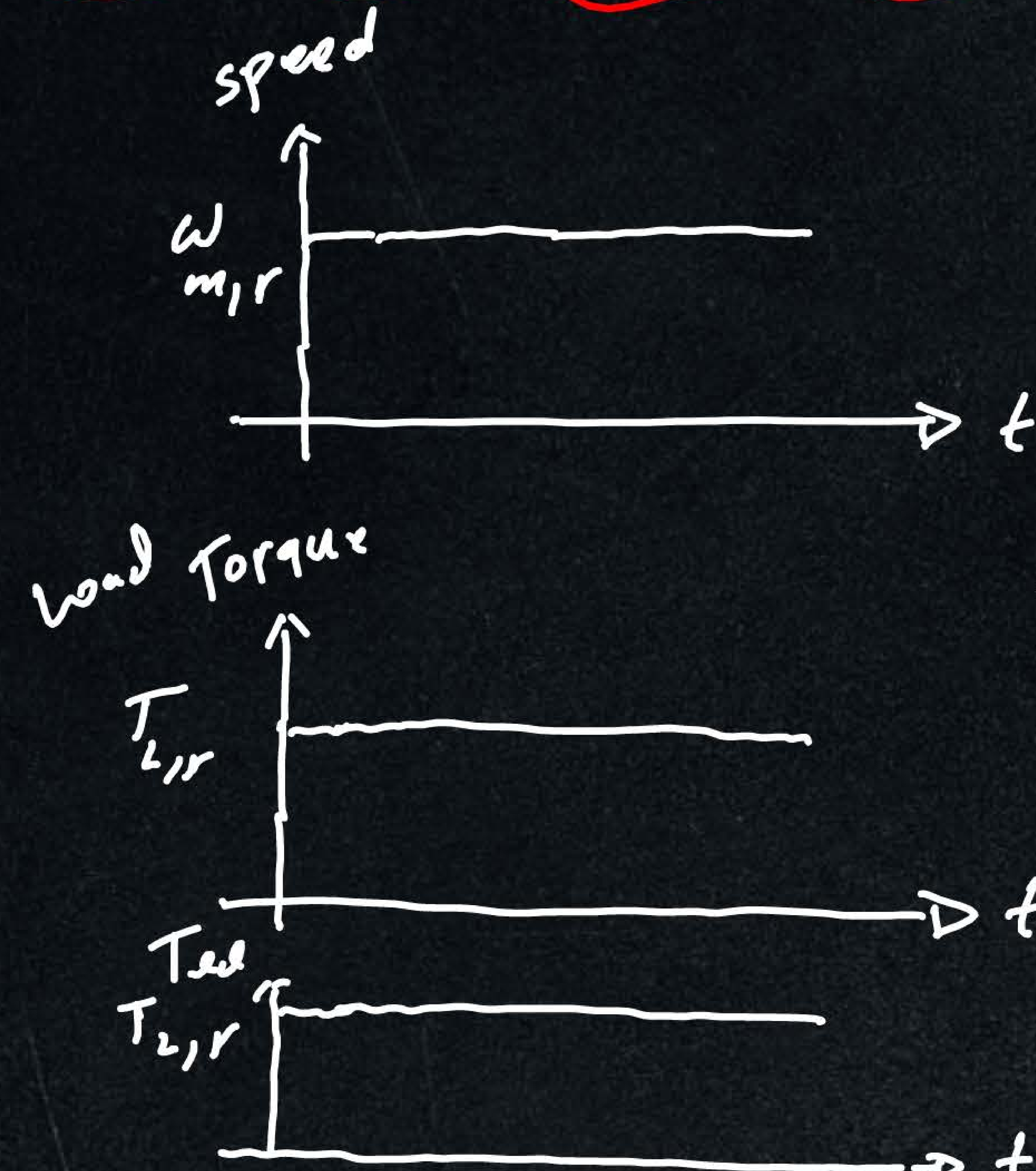
Full-bridge chopper circuit

Four-quadrant chopper circuit

	U_L	i_L	Conducting devices
I	V_s	+	$T_1\ T_2$
	0	+	$T_1\ D_3$ or $T_2\ D_4$
II	$-V_s$	+	$D_3\ D_4$
	0	+	$T_1\ D_3$ or $T_2\ D_4$
III	$-V_s$	−	$T_3\ T_4$
	0	−	$T_3\ D_1$ or $T_4\ D_2$
IV	V_s	−	$D_1\ D_2$
	0	−	$T_3\ D_1$ or $T_4\ D_2$

Calculation of motor's rated torque and power

- constant speed



$\omega_{m,r}$: Rated speed

$T_{L,r}$: Rated torque

$$T_{em} = T_L + J\frac{d\omega_m}{dt}$$

under constant speed operation

$$\frac{d\omega_m}{dt} = 0$$

$$T_{em} = T_L$$

Rated power $= T_{L,r}\,\omega_{m,r}$

rad/sec

$$\omega_{m,r} = n_{m,r} \times \frac{2\pi}{60}$$

rated speed [rad/s] rated speed in [rpm]

• Variable speed

$$T_{el} = T_l + J\frac{d\omega_m}{dt}$$

"Newton's second Law"

speed in r/sec

electro-magnet torque

Load torque

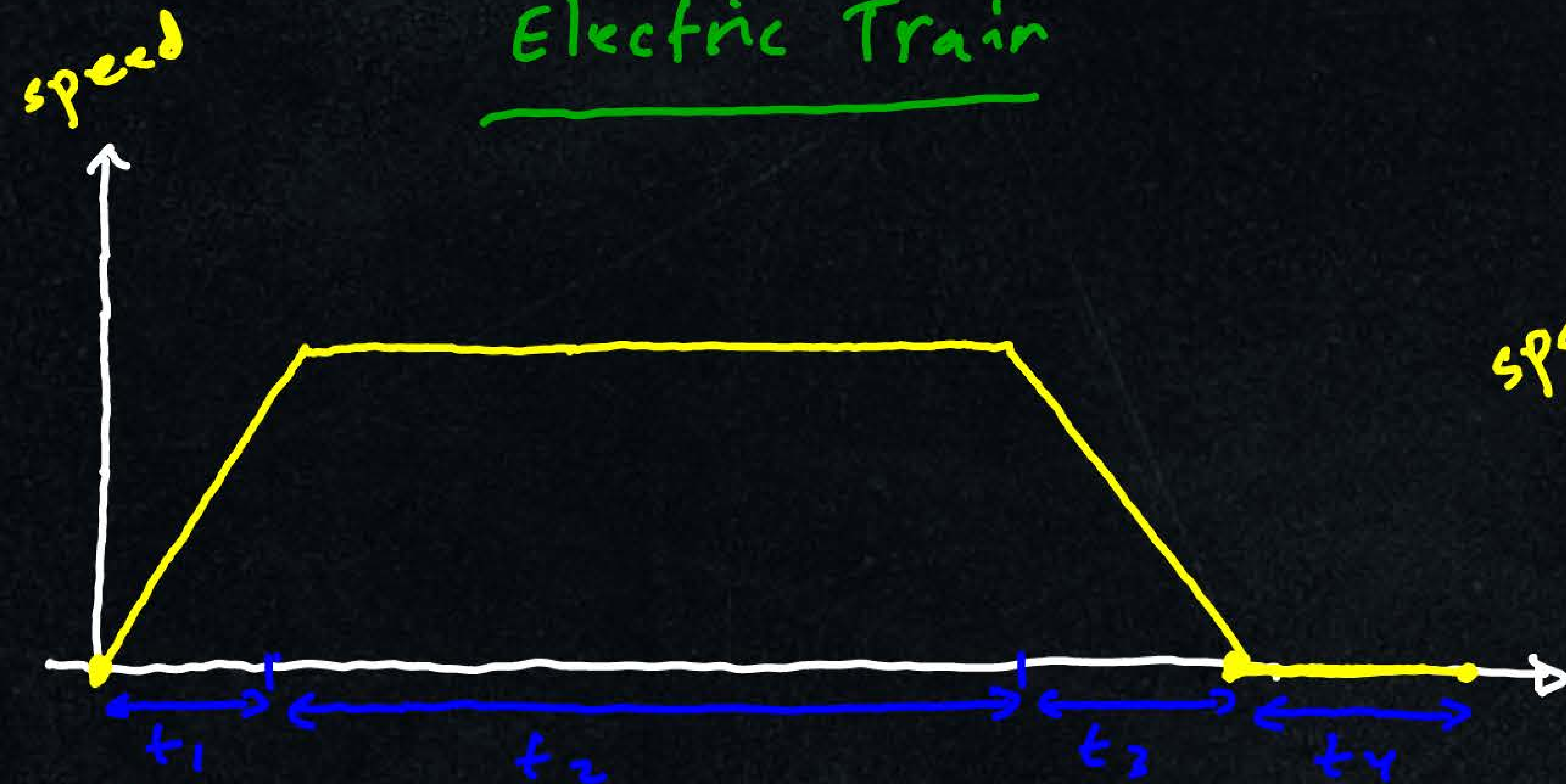
total inertia seen by the motor

$$T_l = T_L + T_W + T_F$$

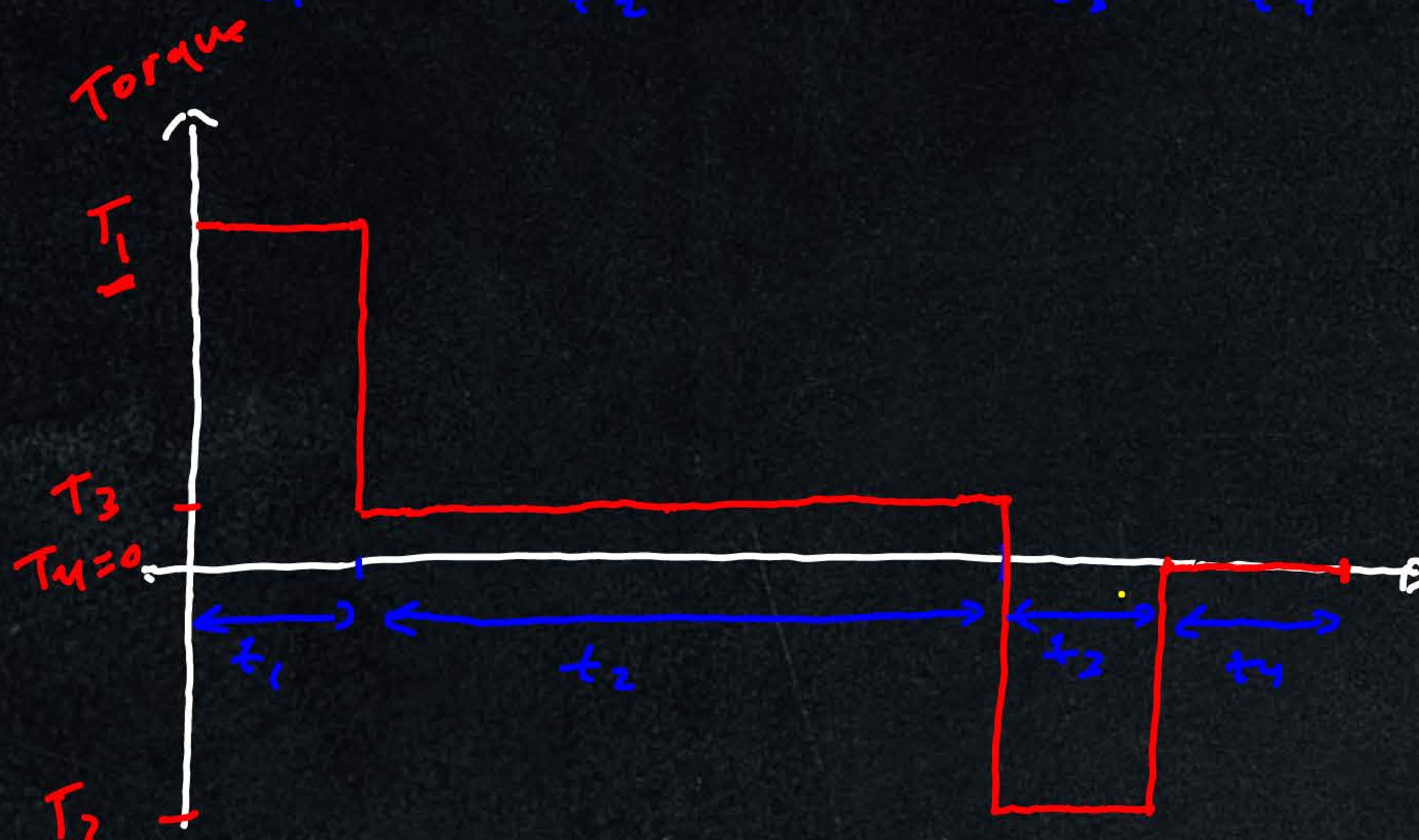
windage torque

Friction torque

Electric Train



speed profile



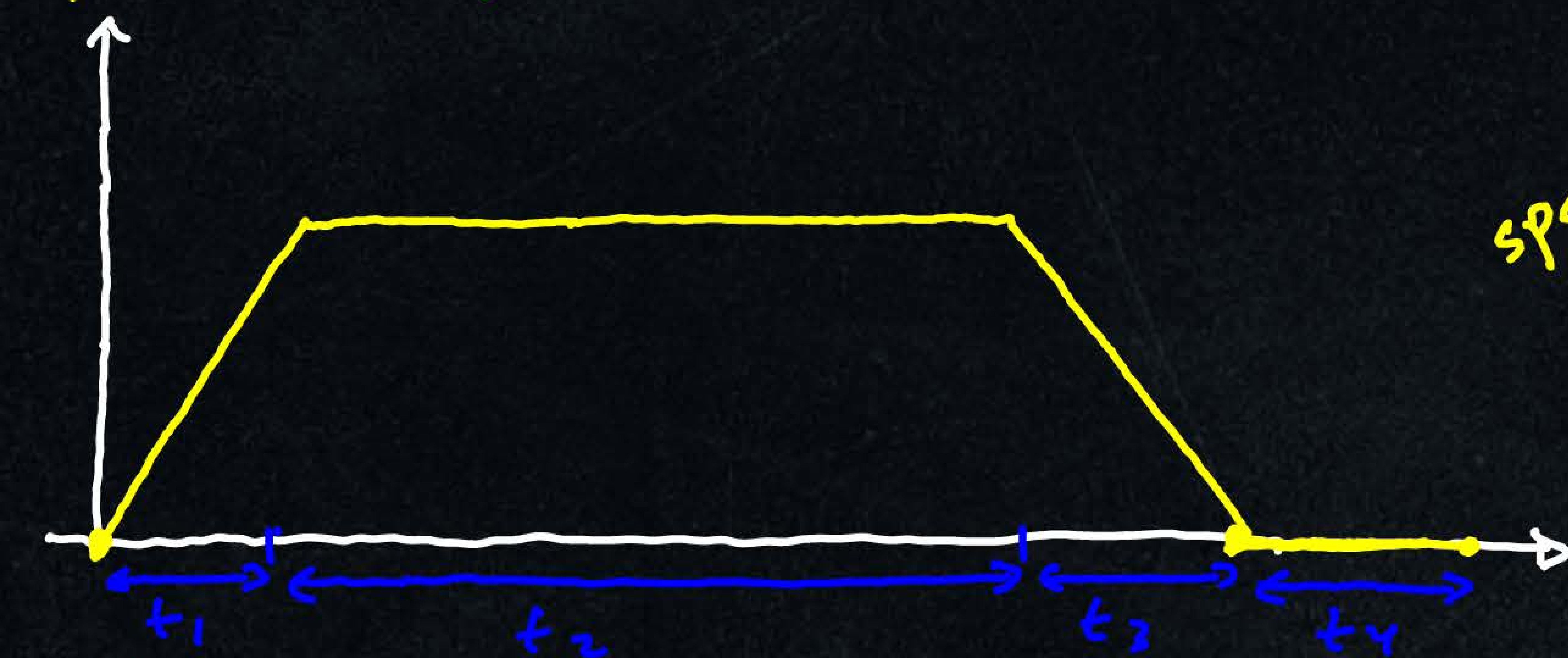
Torque profile

−100 N.m

10 N.m

$$T_{d} = T_l + J\frac{d\omega_m}{dt}$$

Electric Train



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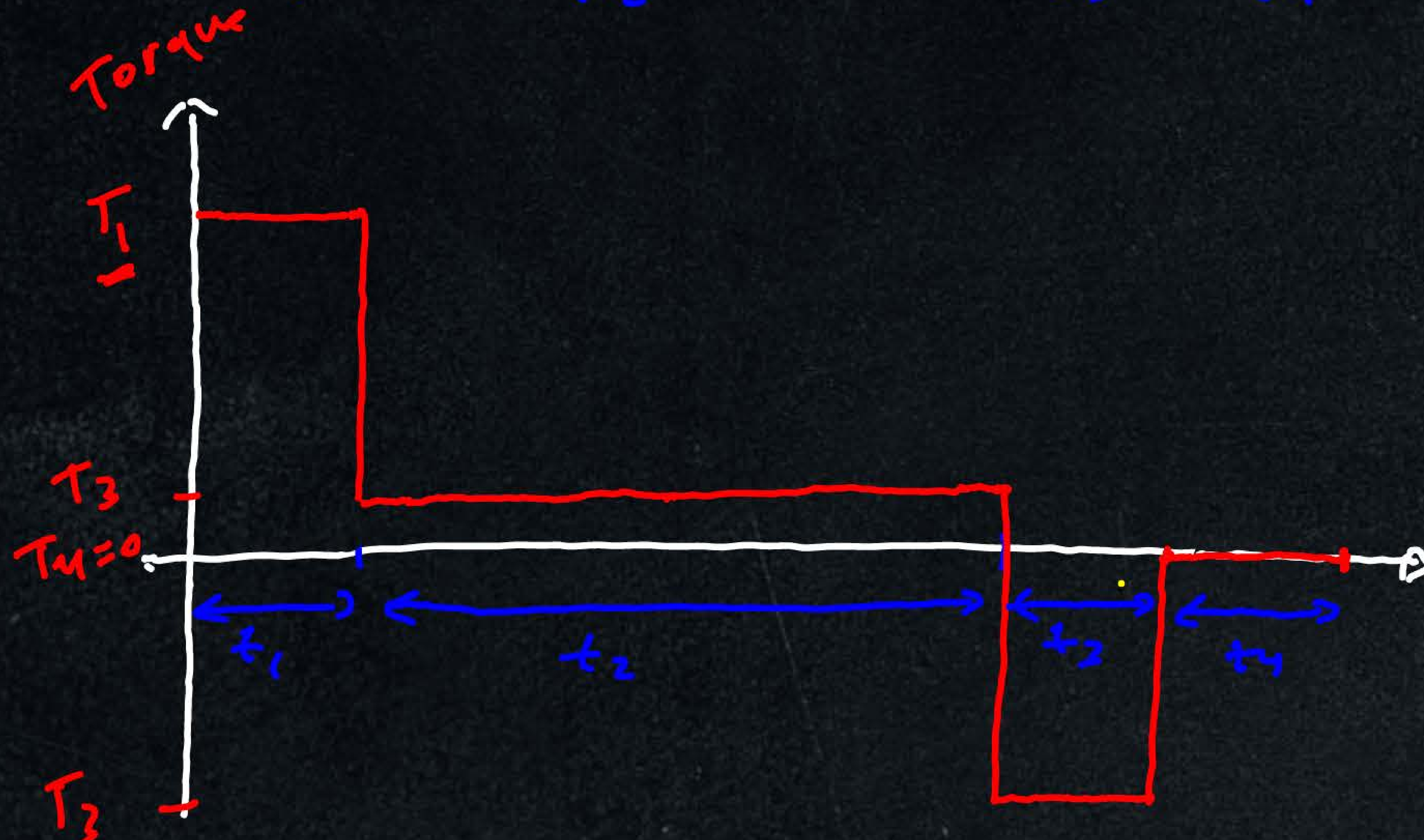


Torque profile

-100 N.m

10 N.m

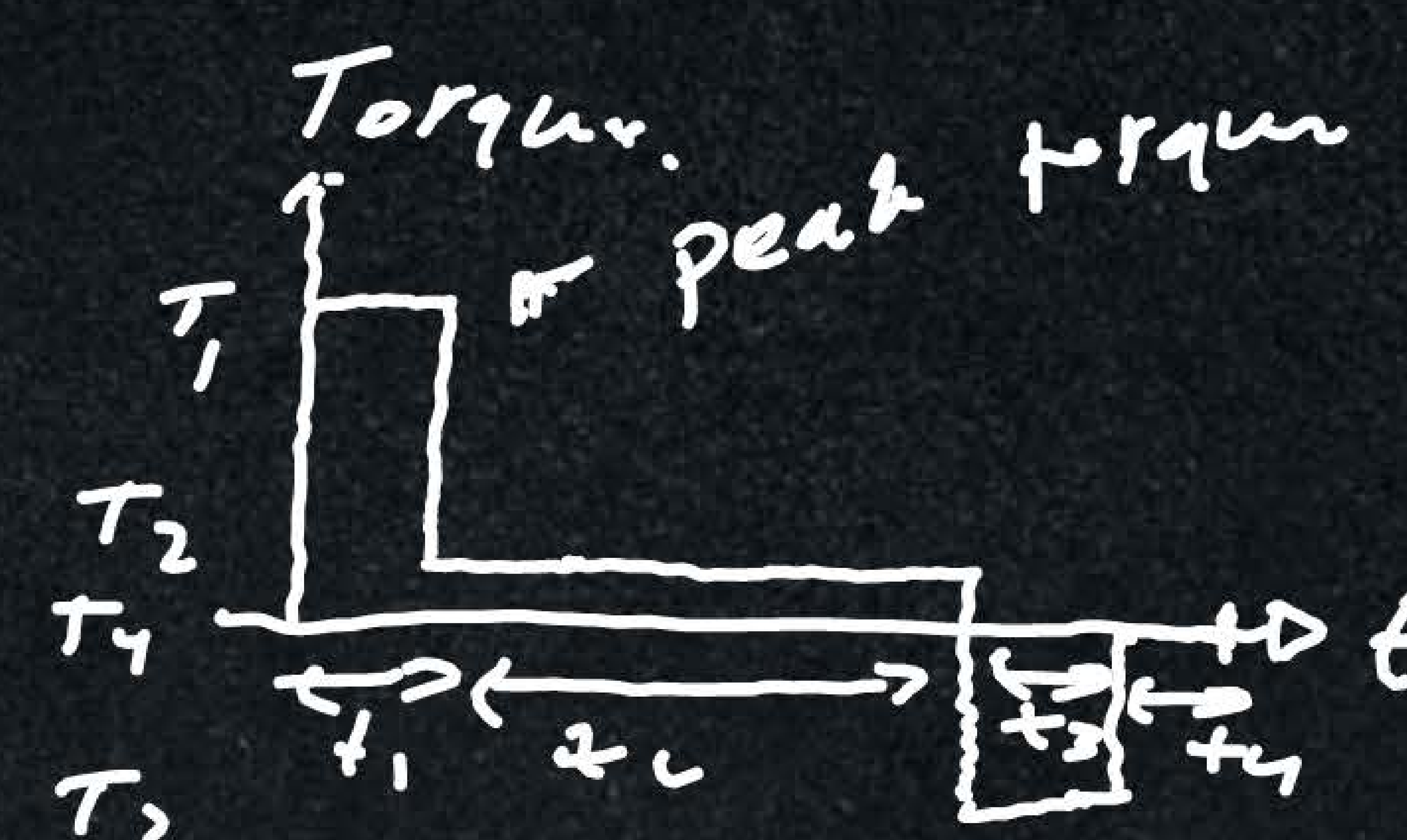
$$T_d = T_l + J\frac{d\omega_m}{dt}$$

The rated torque is calculated using the concept of effective value :-

$$T_{rated} = T_{eff} = \sqrt{\frac{\sum_{i=1}^{N} T_i^2 t_i}{\sum_{i=1}^{N} t_i}}$$

where $\sum_{i=1}^{N} t_i$ is the cycle or period.

$$T_{rated} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3}{t_1 + t_2 + t_3 + t_4}}$$ ✓



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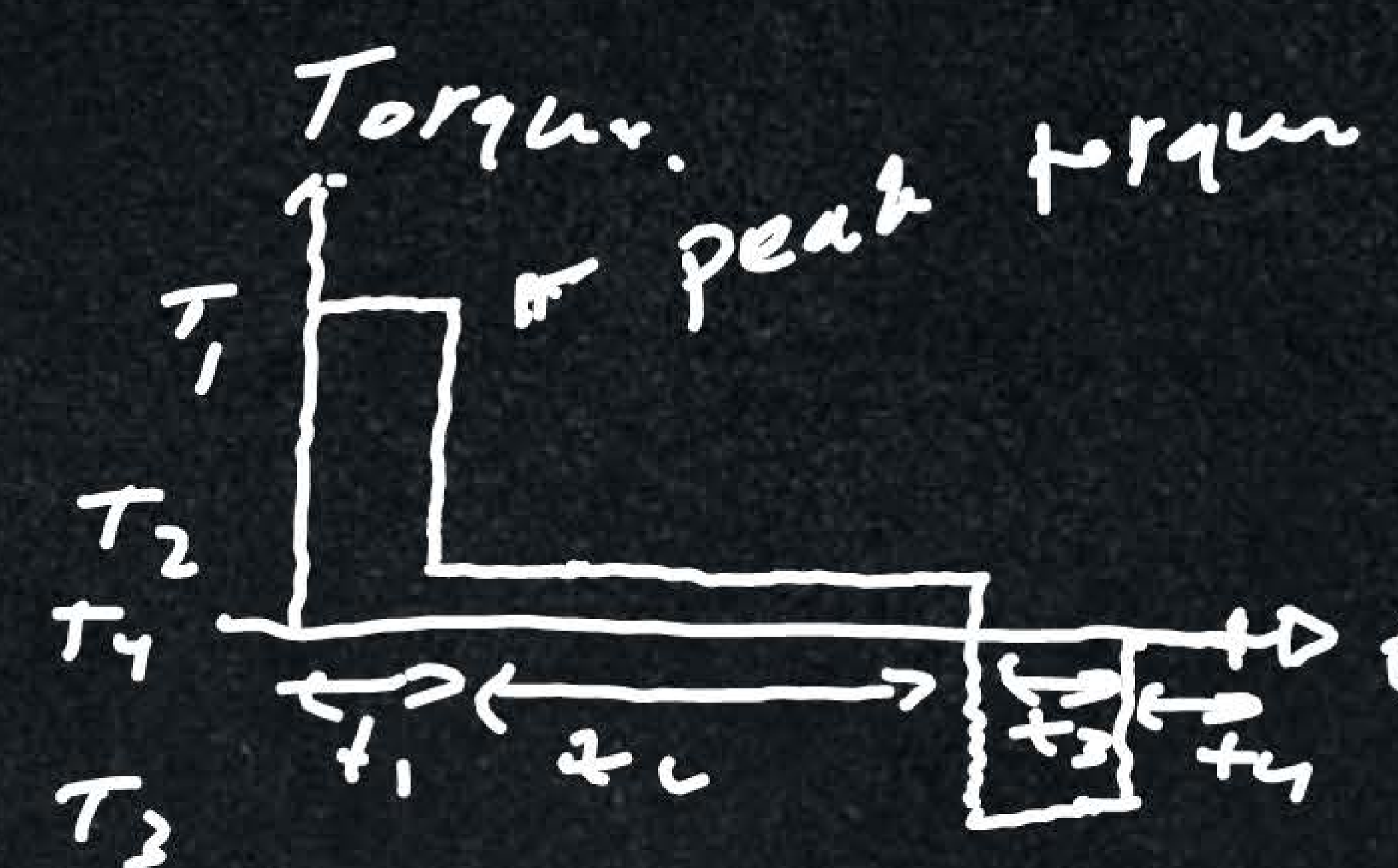

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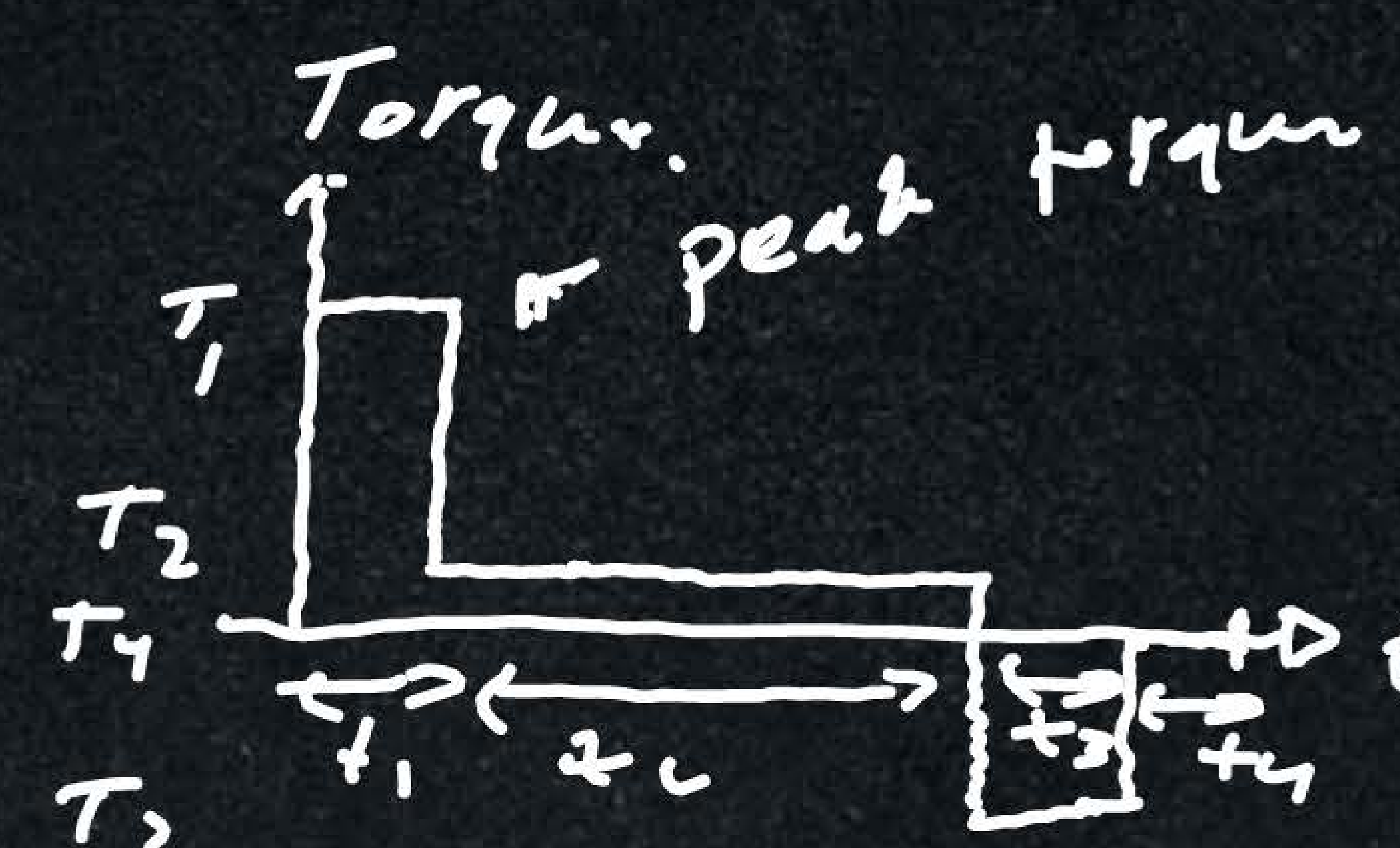
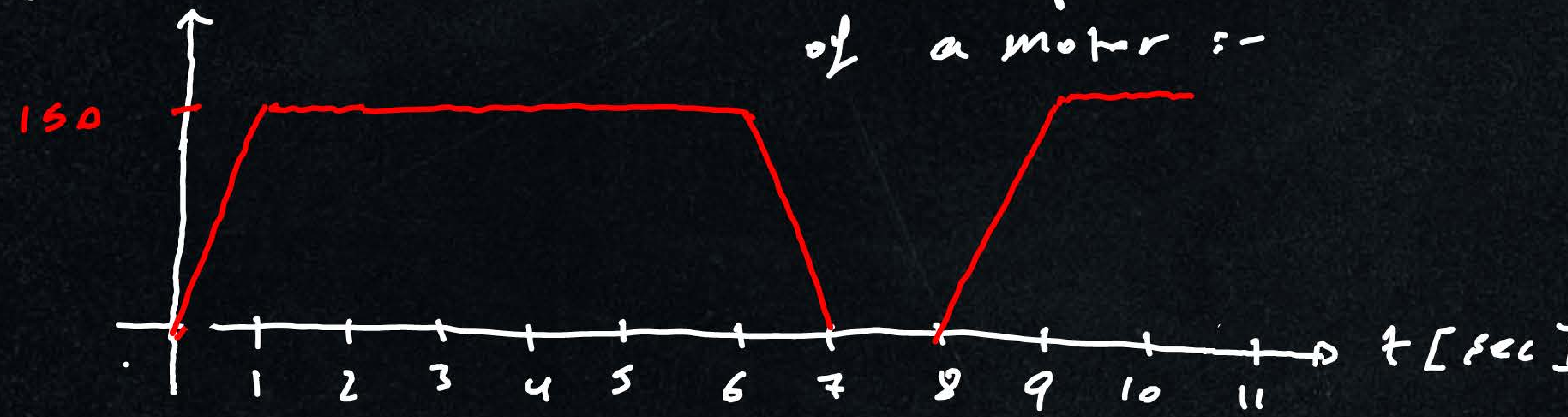
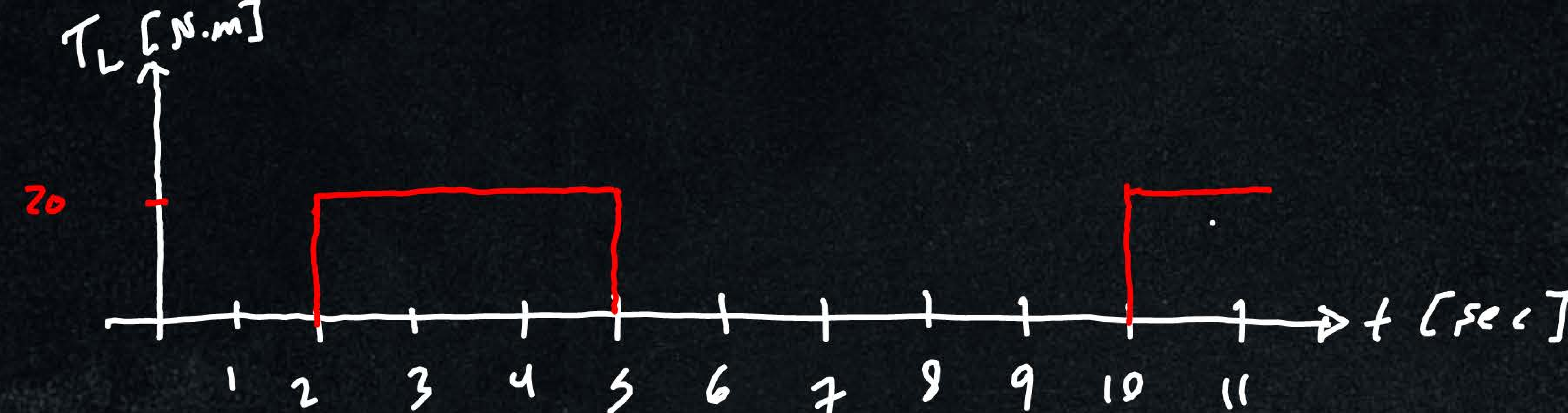


Ex :- ω_m [r/sec]

Given the speed and load torque profiles of a motor :-





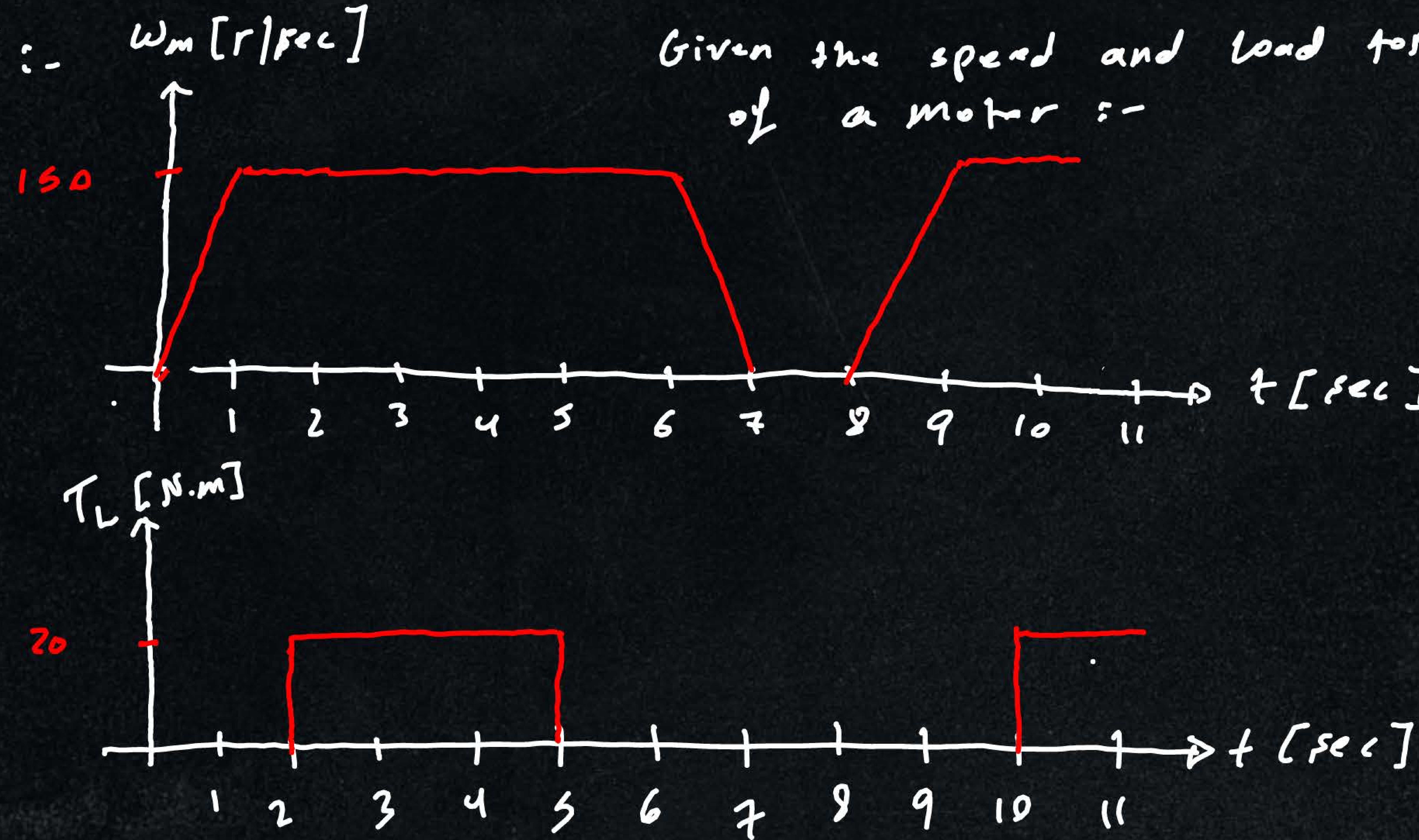
a) Draw the profile of electro-magnet torque, T_{el}.

(Assume that $J = 0.04\ kg.m^2$)

b) Calculate the effective value of T_{el} ?

Ex :-

Given the speed and load torque profiles of a motor :-

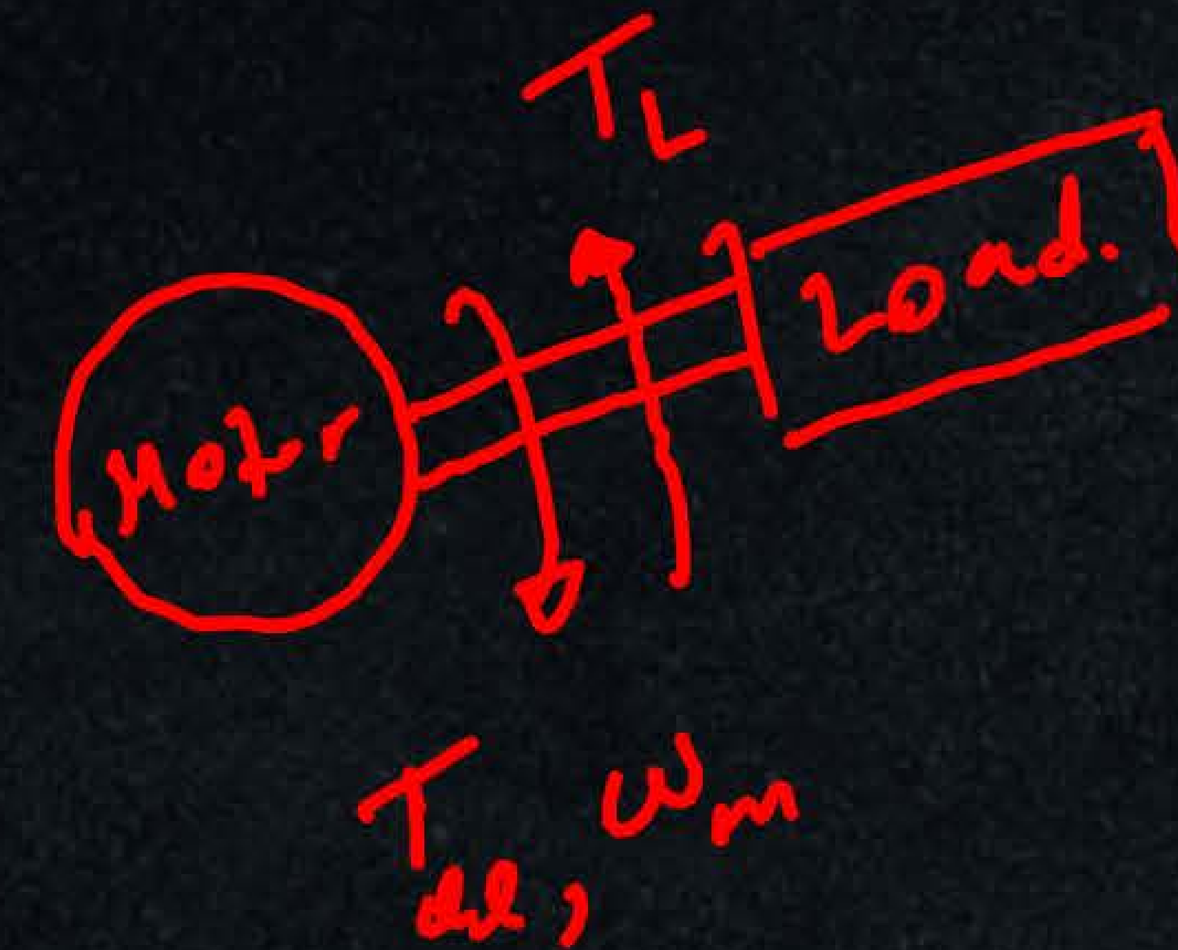


a) Draw the profile of electro-magnet torque, T_{el}.

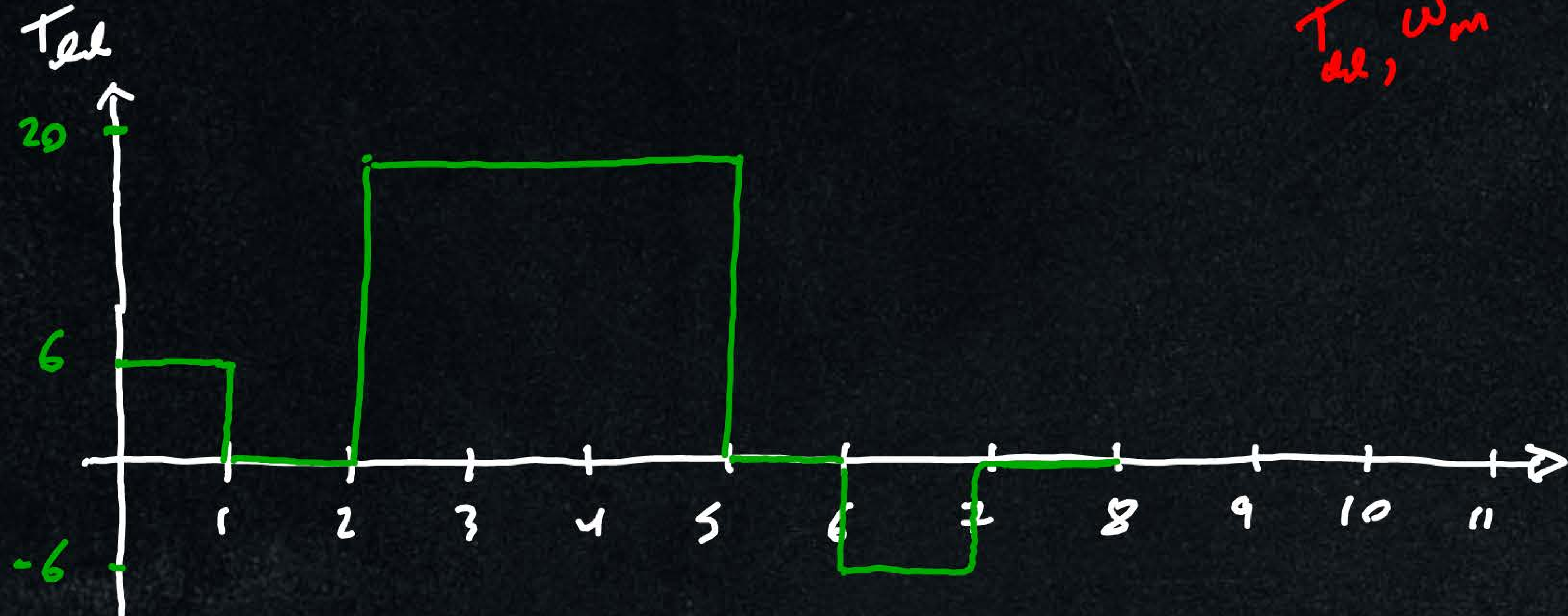
(Assume that $J = 0.04 \ kg.m^2$)

b) Calculate the effective value of T_{el} ?

$$T_{el} = T_L + J\frac{d\omega_m}{dt}$$



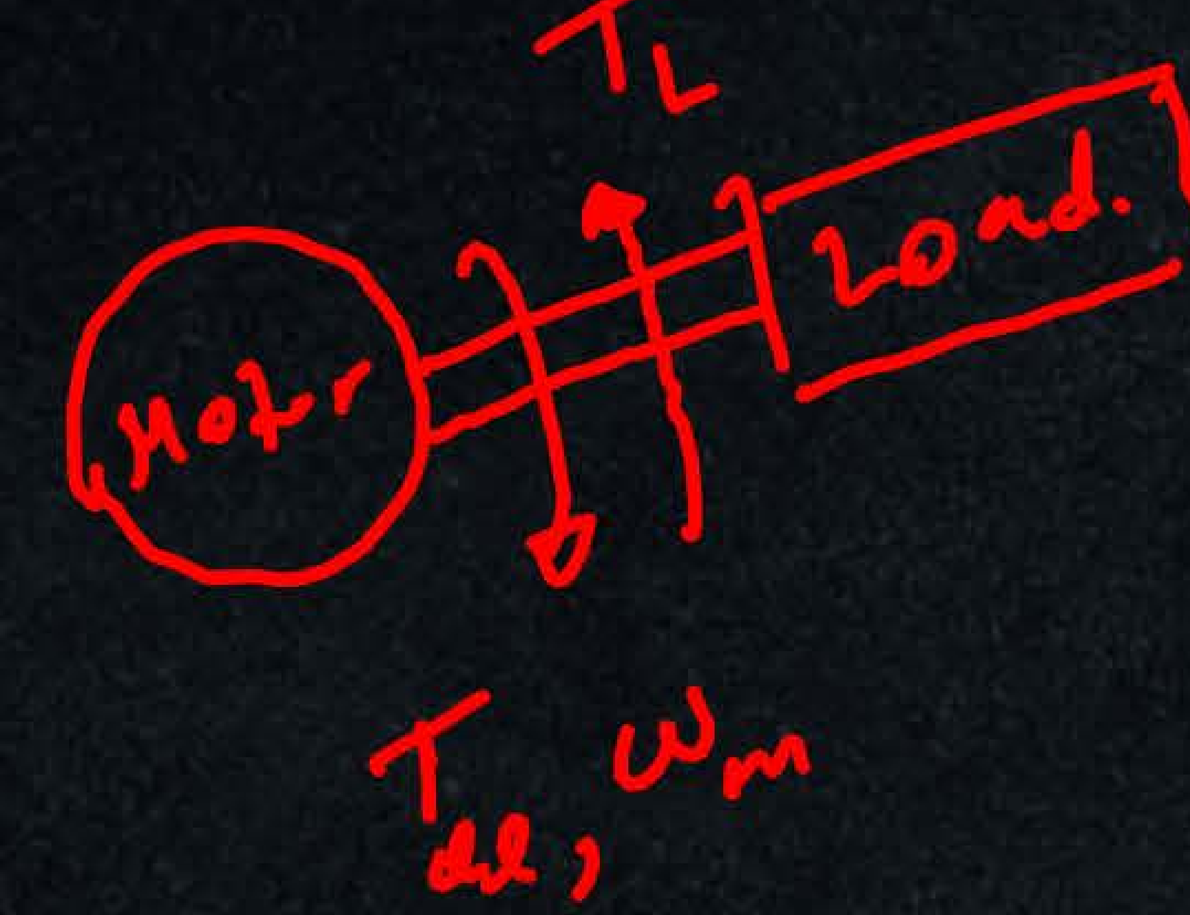
a)



b)

$$T_{eff} = \sqrt{\frac{6^2(1) + 0^2(1) + 20^2(3) + 0^2(1) + (-6)^2(1) + 0^2(1)}{8}} = 12.6 \text{ N.m}$$

$$T_{el} = T_L + J\frac{d\omega_m}{dt}$$



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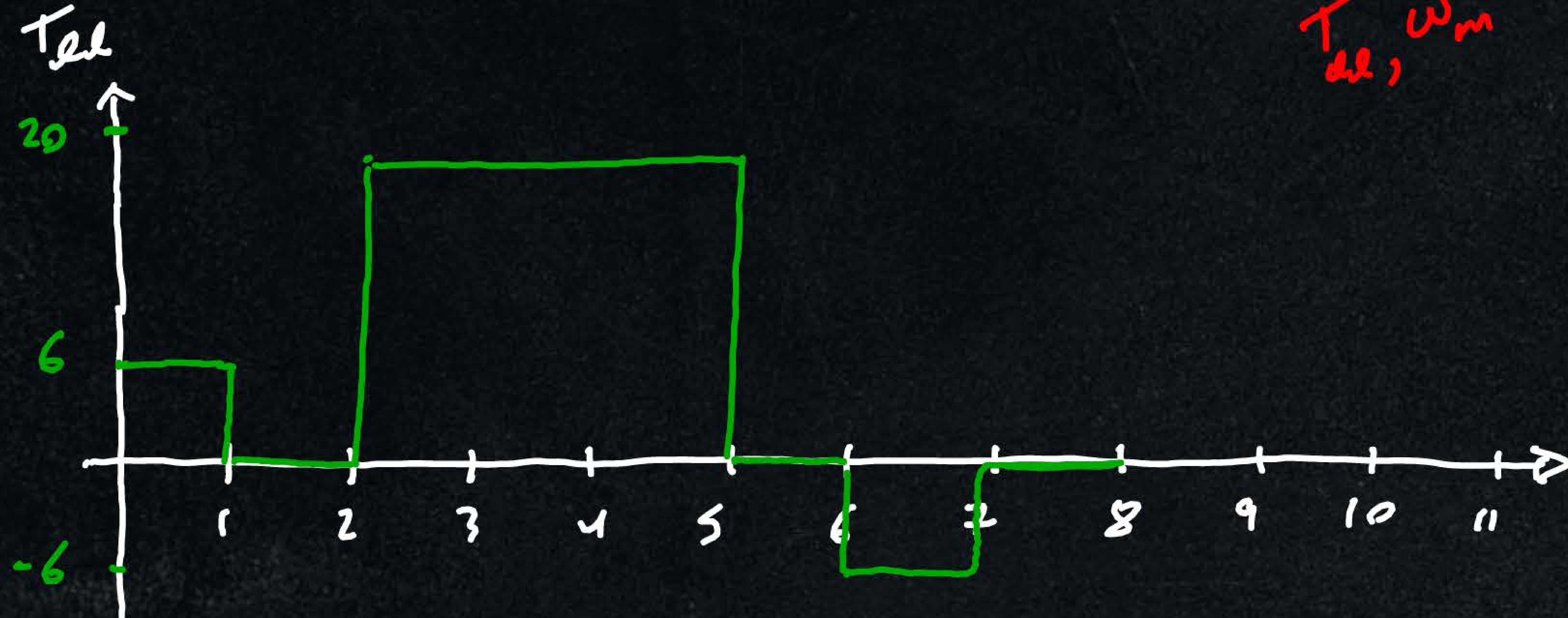

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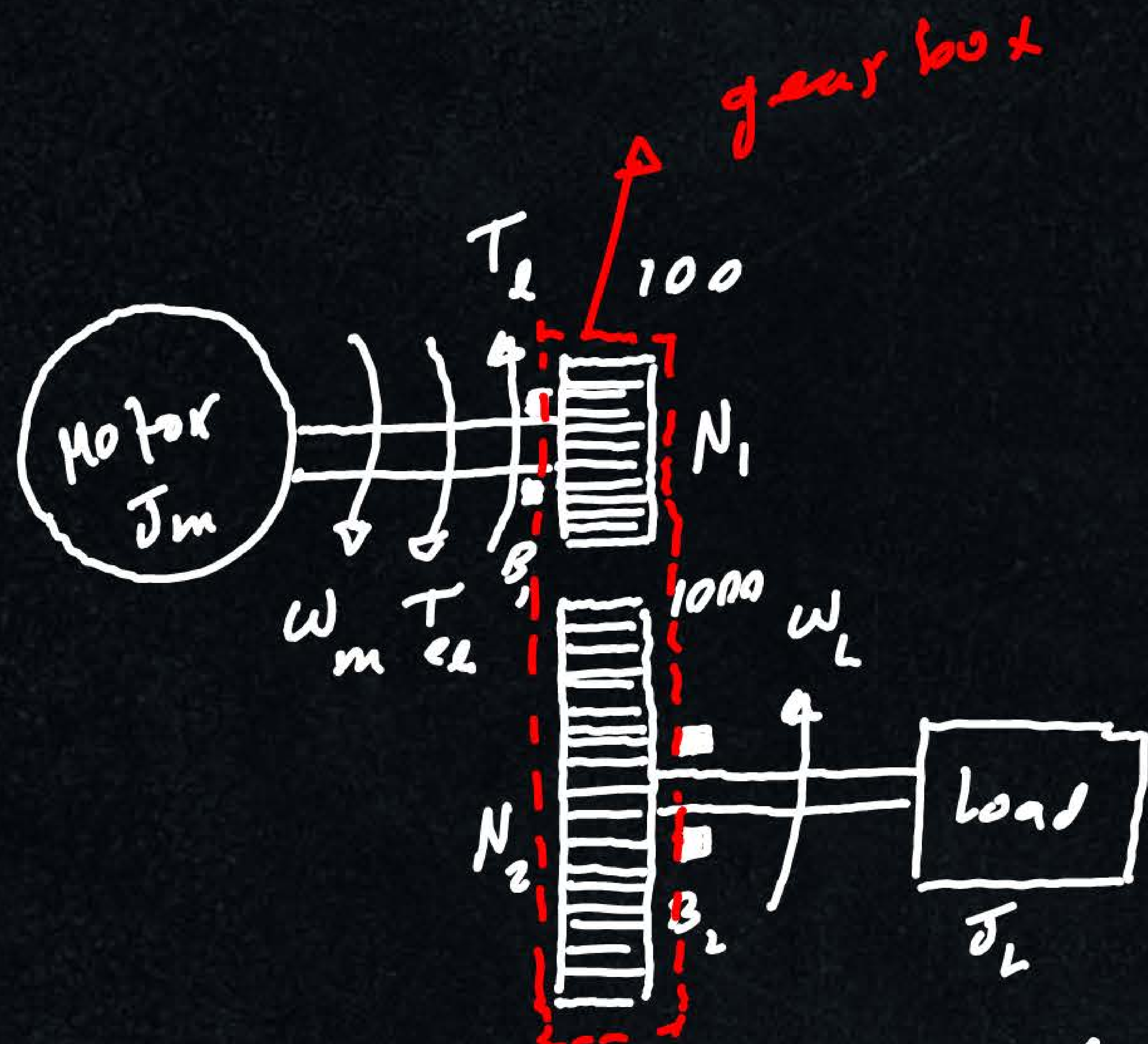


N_1 : Teeth number (motor side)

N_2 : Teeth number (load side)

J_m : Motor's shaft inertia

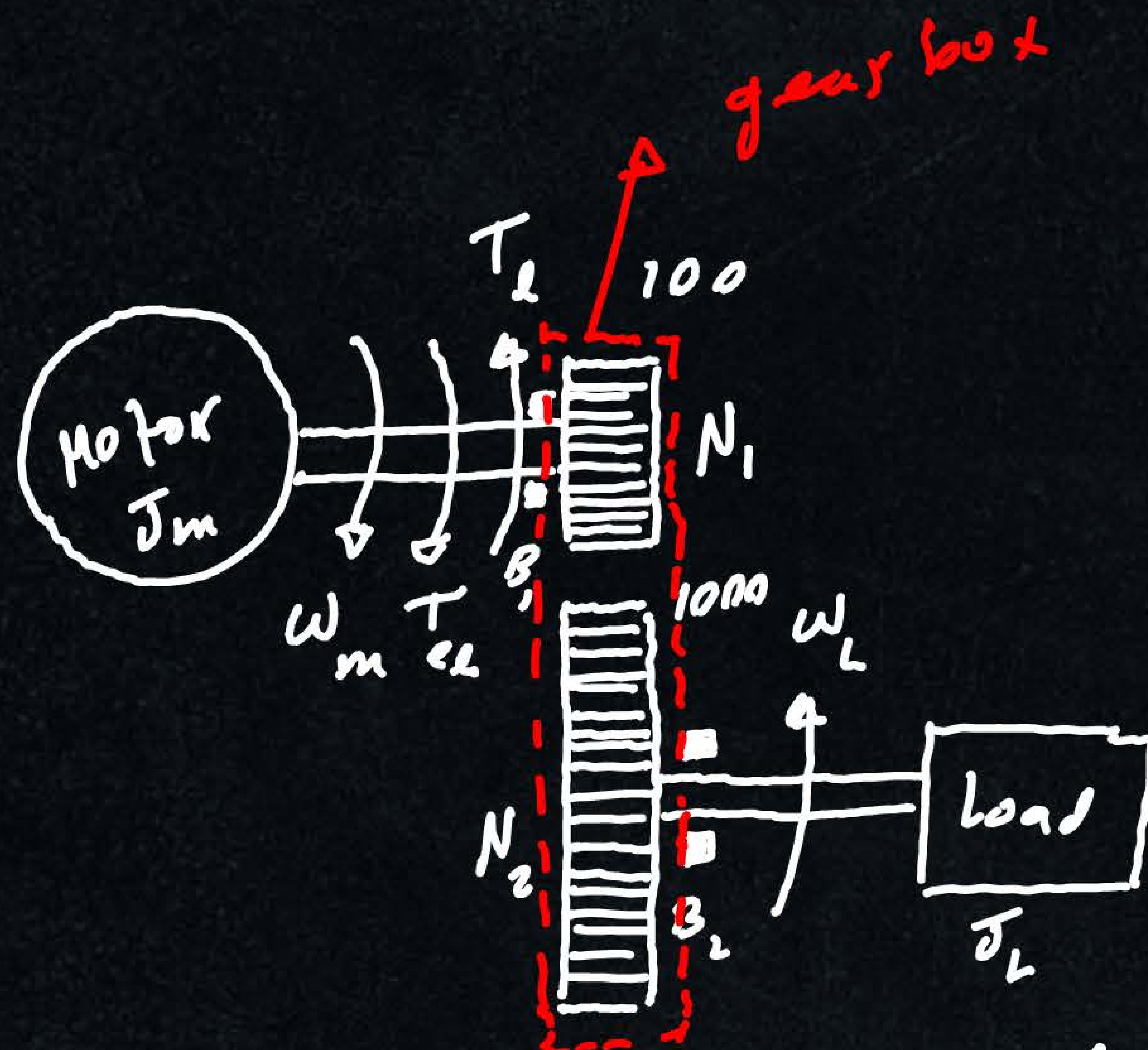
J_L : Load inertia.

B_1, B_2 : Bearings and their friction coefficient.

$$\text{speed} \propto \frac{1}{N}$$

$$\text{speed} \times N = \text{constant}$$

$$\omega_m N_1 = \omega_L N_2 \Rightarrow \frac{\omega_m}{\omega_L} = \frac{N_2}{N_1} = \tau \; ; \quad \tau : \text{gear box ratio.}$$



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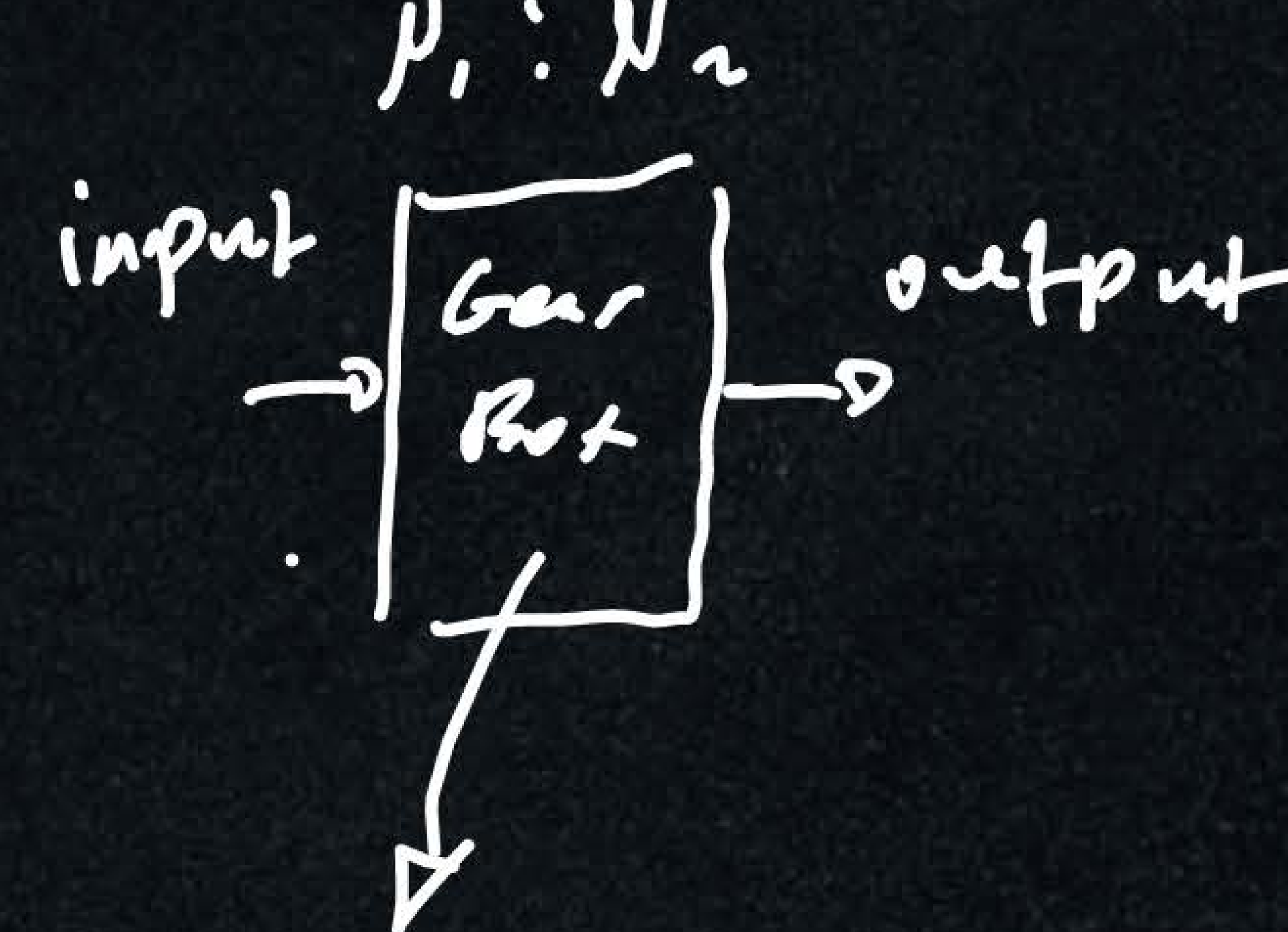
$$\omega_m N_1 = \omega_L N_2 \Rightarrow \boxed{\frac{\omega_m}{\omega_L} = \frac{N_2}{N_1} = \tau} \quad (1)$$

; τ : gear-box ratio.

Assume that the power handled by the gear-box is the same on both sides

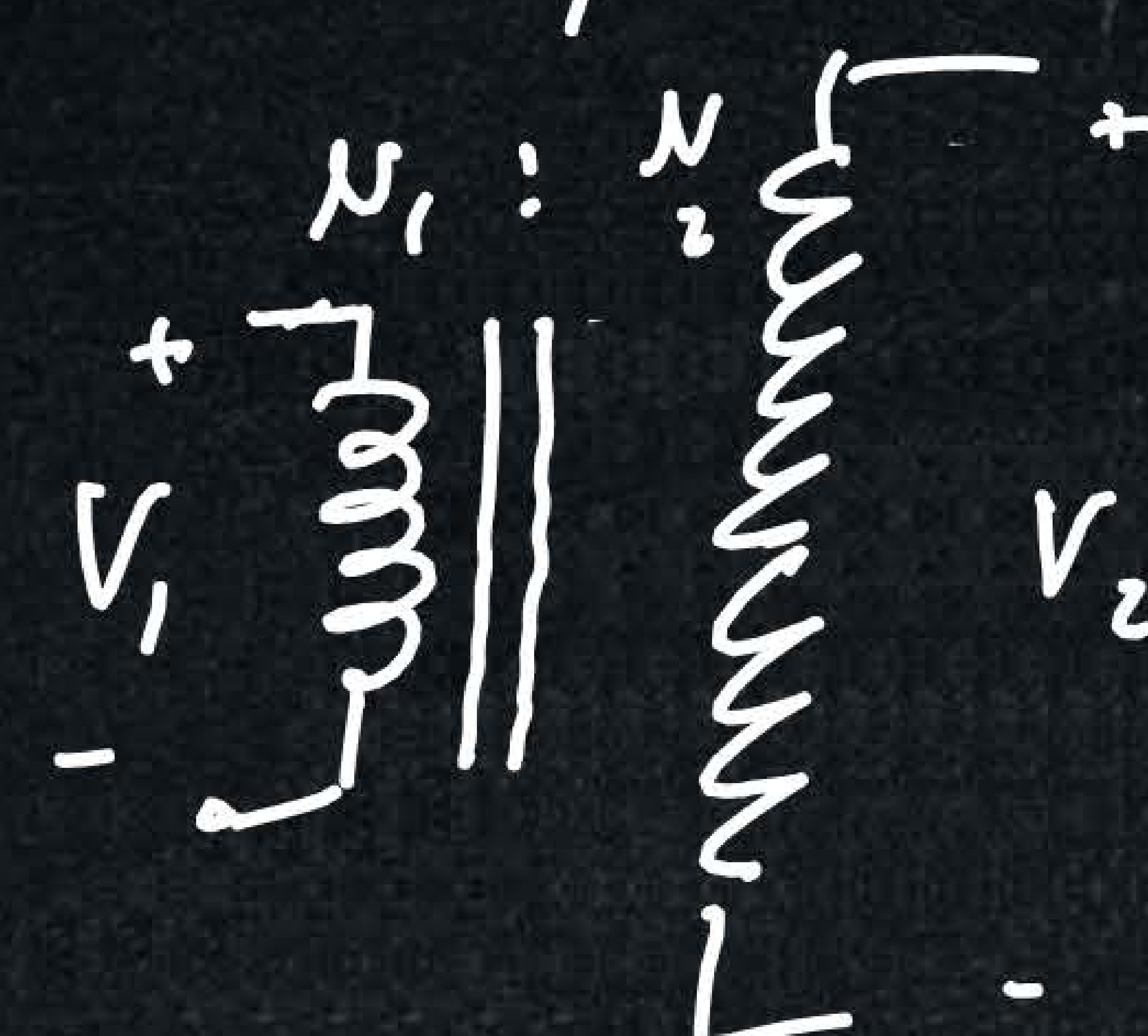
$$P_{input} = P_{output}$$



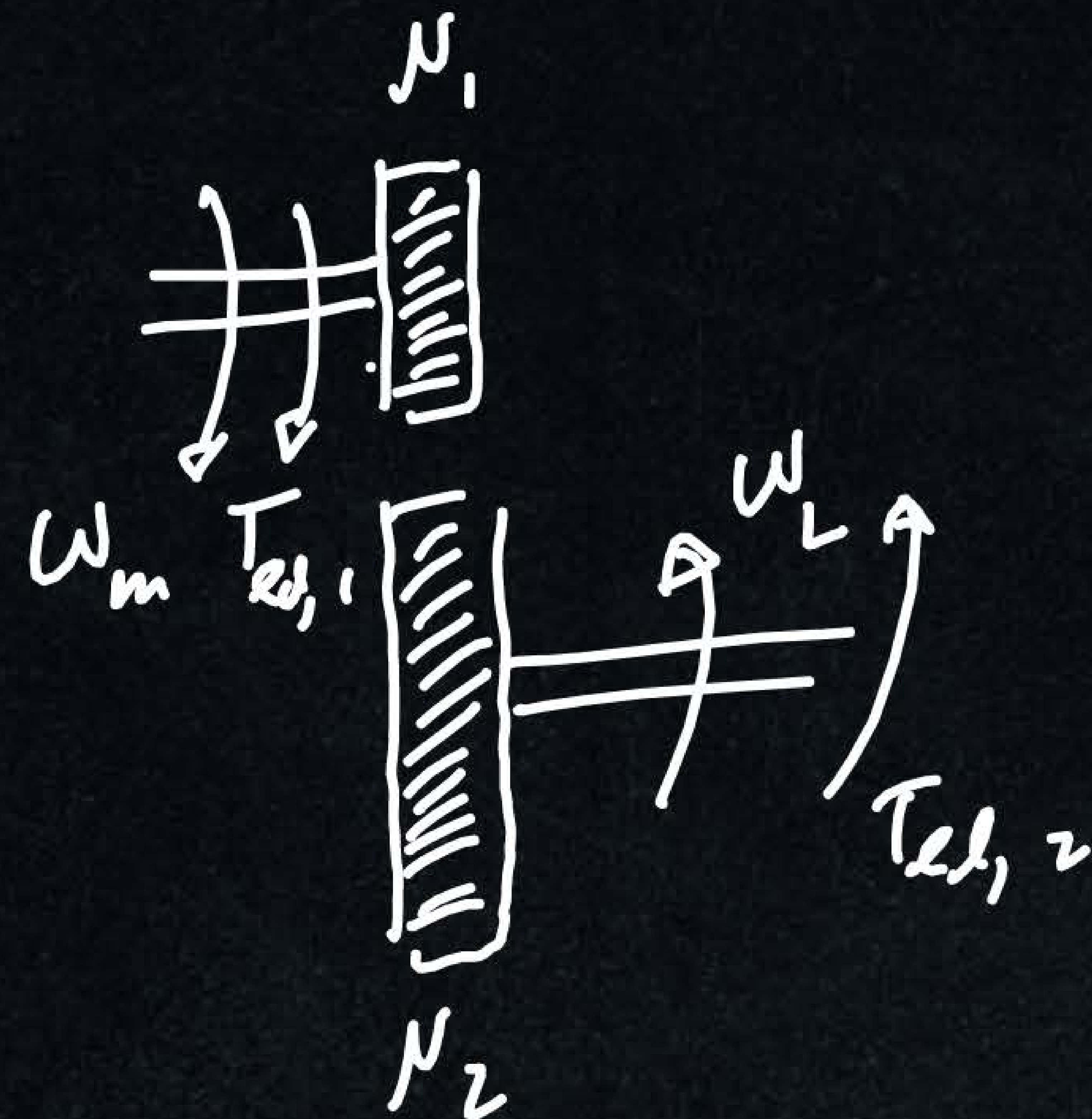
$$T_{el,1}\,\omega_m = T_{el,2}\,\omega_L$$

$$\boxed{\frac{T_{el,1}}{T_{el,2}} = \frac{\omega_L}{\omega_m} = \frac{1}{\tau}} \quad \cdots (2)$$

Torquer Transformer



Gear box model (ideal model)



$$\tau = \frac{N_2}{N_1}$$

$$\frac{\omega_m}{\omega_L} = \tau \quad \text{---} \ 1$$

$$\frac{T_{el,1}}{T_{el,2}} = \frac{1}{\tau} \quad \text{---} \ 2$$

Reflected inertia

Torque developed by the motor, $T_{el,1}$, is :-

$$T_{el,1} - T_{l} = J \frac{d\omega_m}{dt}$$

where J is the total inertia seen by the motor, which is given by:

$$J = J_m + J_{ref}$$

J_m: inertia of shaft

J_{ref}: reflected inertia to the motor shaft

$$\cancel{\frac{1}{2}} J_{ref}\,\omega_m^2 = \cancel{\frac{1}{2}} J_L\,\omega_L^2$$

$$J_{ref}\,\omega_m^2 = J_L\,\omega_L^2$$

$$J_{ref} = J_L\left(\frac{\omega_L}{\omega_m}\right)^2 \quad ; \quad \frac{\omega_L}{\omega_m} = \frac{1}{\tau}$$

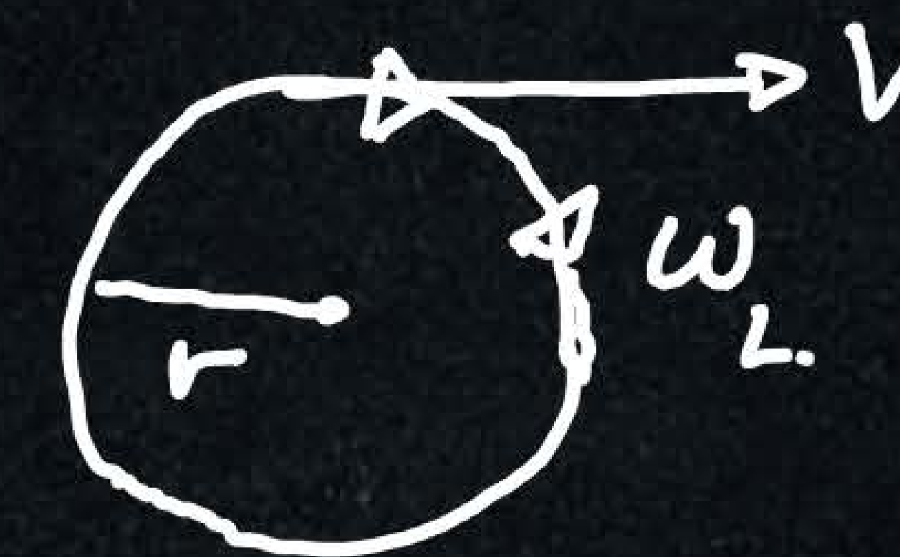
$$\boxed{J_{ref} = \frac{J_L}{\tau^2}}$$

$\therefore$ The total inertia is given by :

$$\boxed{J = J_m + \frac{J_L}{\tau^2}}$$

Calculation of J_L for electric vehicle (EV):-

$$\cancel{\frac{1}{2}} J_L \omega_L^2 = \cancel{\frac{1}{2}} m V^2$$



$$J_L = \frac{m V^2}{\omega_L^2}$$

$$V = \omega_L r$$

$$J_L = m \left(\frac{\cancel{\omega_L} r}{\cancel{\omega_L}} \right)^2$$

$$\boxed{J_L = m r^2}$$

m : mass in [kg]

r : radius in [m]

W_L : speed of wheel in [r/s]

V : Linear speed in [m/s]

$$J_{ref} = J_L / \tau^2 = m \left(\frac{r}{\tau} \right)^2$$

- Maximum power transfer to the load occurs if the load inertia reflected to the motor shaft, J_{ref}, equals the motor inertia, J_m. In this case, maximum acceleration of the load will result.

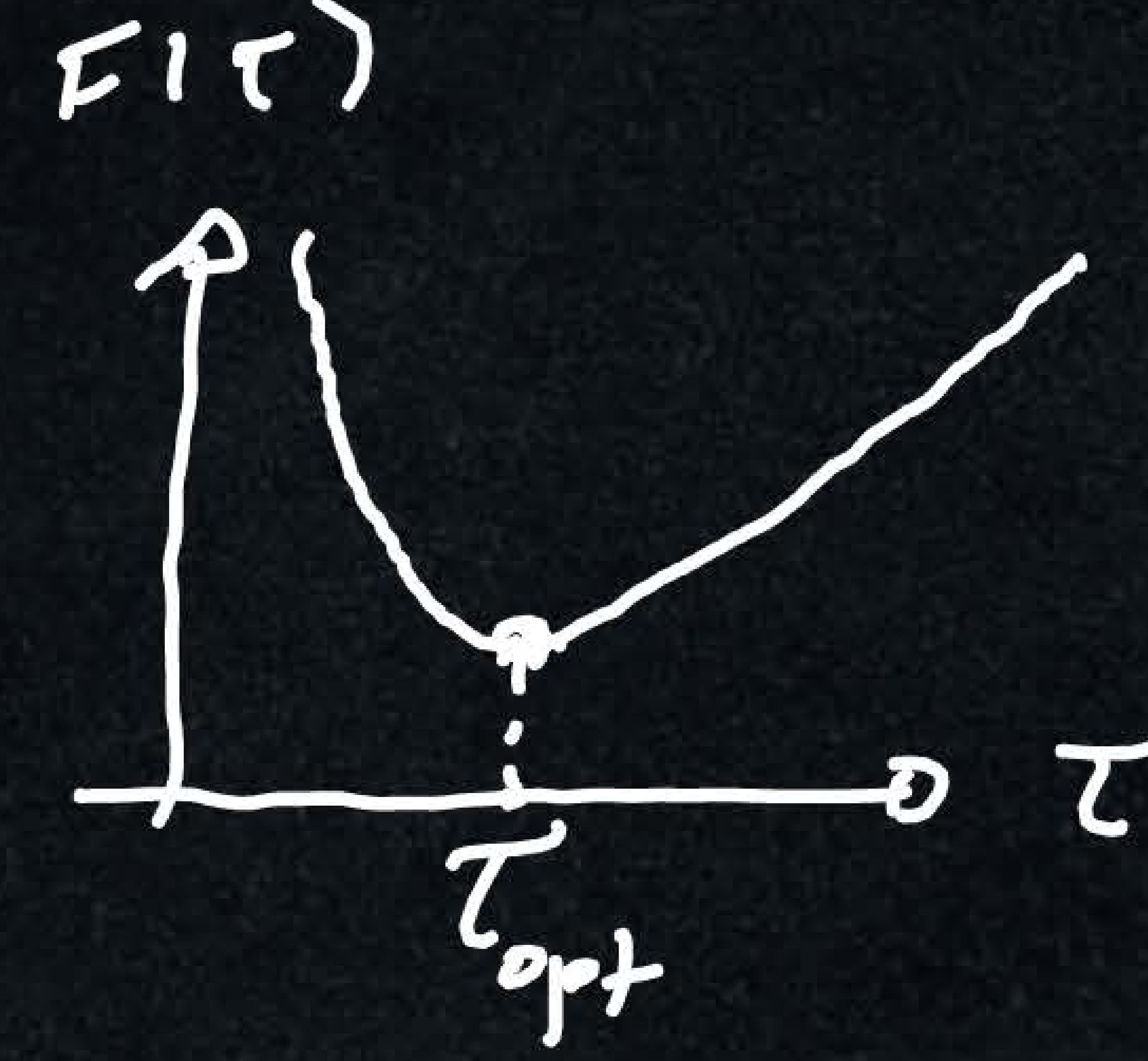
$\Rightarrow$ load inertial optimization.

<u>proof</u> :- The torque developed by the motor is:-

$$T_{el,1} = T_l + J\frac{d\omega_m}{dt} \Rightarrow \Sigma T = \left(J_m + J_{ref}\right)\frac{d\omega_m}{dt}, \quad \omega_m = \tau\,\omega_L$$

$$\Sigma T = \left(J_m + \frac{J_L}{\tau^2}\right)\frac{d(\tau\omega_L)}{dt} = \left(\tau J_m + \frac{J_L}{\tau}\right)\frac{d\omega_L}{dt} = \left(\tau J_m + \frac{J_L}{\tau}\right)\underbrace{\alpha_L}_{\text{Load acceleration}}$$

$$\frac{\varepsilon T}{\alpha_L} = J_m \tau + \frac{J_L}{\tau} = \underline{F(\tau)}$$



$$\min(F(\tau)) \Rightarrow \max(\alpha_L)$$

$$\frac{\partial F(\tau)}{\partial \tau} = 0 \Rightarrow J_m - \frac{J_L}{\tau^2} = 0$$

$$J_m = \frac{J_L}{\tau^2} \Rightarrow \boxed{J_m = J_{ref}}$$

$$\tau = \sqrt{J_L / J_m}$$

Nissan Leaf

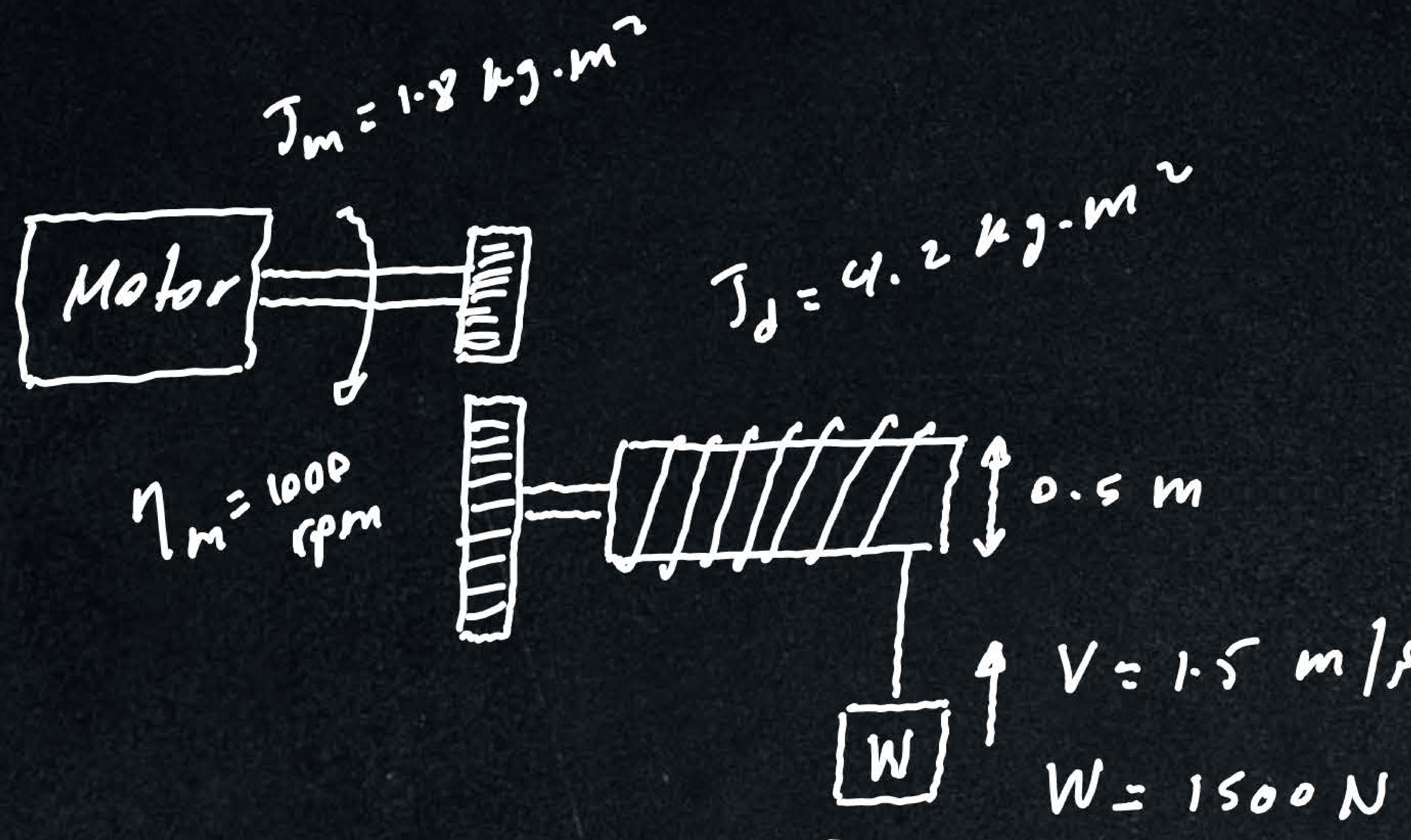
$J_m = 1.5$

$J_{ref} = 2.8 - 3$

0 – 100 km/h. τ.

9.7 sec $\boxed{10}$

EX :- A weight of 1500 N is to be lift up with a velocity of 1.5 m/s. The winch has a diameter of 0.5 m and driven by a motor running at 1000 rpm. The inertia of the motor and the winch drum are 1.8 kg.m^2 and 4.2 kg-m^2, respectively.



1) Calculate the gear-box ratio

$$\tau = \frac{\omega_m}{\omega_L} \; ; \quad \left.\begin{aligned} \omega_m &= 1000 * \frac{\pi}{30} \\ \omega_L &= \frac{V}{r} = \frac{1.5}{(0.5/2)} \end{aligned}\right) \Rightarrow \tau = \frac{1000 \frac{\pi}{30}}{\left(\frac{1.5}{0.75}\right)} = 17.45$$

2) Calculate the reflected inertia to the motor's shaft

$$J_{tot} = J_m + J_{ref}$$

$$J_{ref} = J_d' + \underline{J_L'} \; ; \quad J_d' = \frac{J_d}{\tau^2} = \frac{4.2}{(17.45)^2} = 0.0138$$

$$\frac{1}{2} m v^2 = \frac{1}{2} J_L' \omega_m^2 \Rightarrow \frac{1500}{9.8} (1.5)^2 = J_L' \left(1000 \frac{\pi}{30}\right)^2$$

$$J_L' = 0.03144 \text{ kg.m}^2 \qquad J_{ref} = 0.0452 \text{ kg.m}^2$$

3) Calculate the torque developed by the motor in term of motor's acceleration

$$T_{el} = \underbrace{T_l}_{\downarrow} + J_{tot} \frac{d\omega_m}{dt} \quad ; \quad J_{tot} = J_m + J_{ref} = 1.8 + 0.045 = 1.845 \ kg.m^2$$

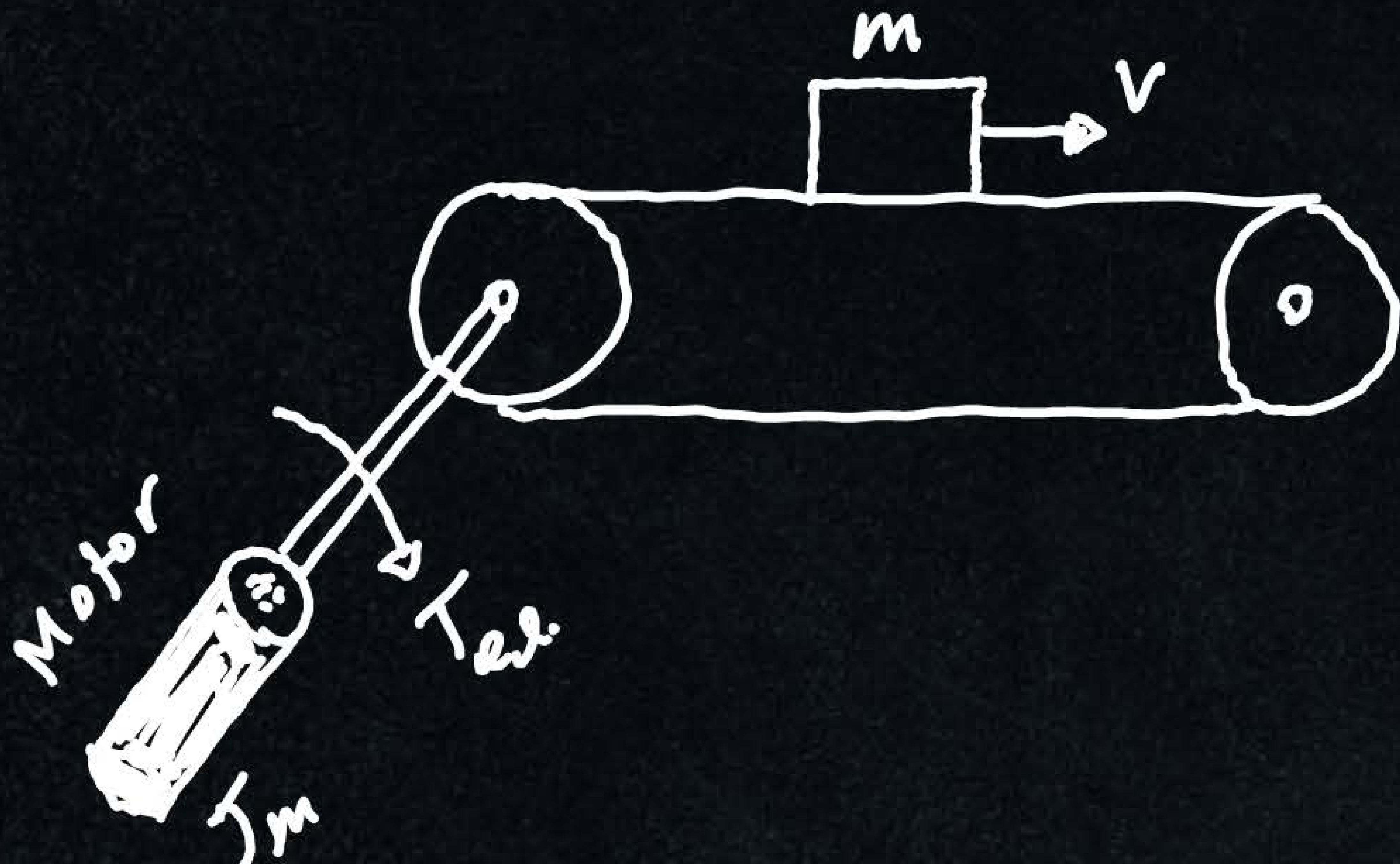
$$T_l = \frac{W r}{\tau} = \frac{1500\left(\frac{0.5}{2}\right)}{17.45} = 21.5 \ N.m$$

$$\boxed{T_{el} = 21.5 + 1.845\,\alpha_m} \ N.m$$

4) Calculate the torque developed by the motor under steady-state operation.

$$T_{el} = 21.5 \ N.m \quad ; \quad \frac{d\omega_m}{dt} = 0$$

Ex:- Consider the belt and pully system shown in Fig. 1.



J_m : motor's inertia = 0.006 kg.m^2

m : mass of load = 0.5 kg

r : pully radius = 0.1 m

1) Calculate the reflected inertia to the motor's shaft

$$\frac{1}{2} m V^2 = \frac{1}{2} J_{ref} \, \omega_m^2$$

$$J_{ref} = m \left(\frac{V}{\omega_m}\right)^2 = m \left(\frac{\cancel{\omega_m} r}{\cancel{\omega_m}}\right)^2 = m r^2 = 0.5 (0.1)^2$$

$$J_{ref} = 5 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

b) Calculate the total inertia seen by the motor

$$J_{tot} = J_m + J_{ref} = 0.006 + 0.005 = 0.011 \ kg\cdot m^2$$

c) Calculate the torque required from the motor to accelerate the load from rest to a velocity of 1 m/s in a time of 3 sec.

$$T_{al} = \cancel{T_l}^{\,0} + J_{tot}\frac{d\omega_m}{dt} \Rightarrow T_{al} = 0.011\,\frac{\Delta\left(\frac{V}{r}\right)}{\Delta t} = 0.011\,\frac{\left(\frac{1}{0.1}\right)}{3}$$

$$T_{al} = \frac{0.011}{0.3} = 0.0367 \ N.m \ .$$